MEAN AND EDDY DYNAMICS OF THE MAIN THERMOCLINE

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Abstract.

This paper reviews and discusses a selection of developments in the theory of the structure of the main thermocline, and the mesoscale eddies that inhabit it. In classical theories, that is theories that assume a steady, near-laminar flow and that are based on the planetary geostrophic equations, the upper thermocline (below the surface mixed layer) is conservative and advectively dominated — that is, the dominant balance in the thermodynamic equation lies in the advective terms, leading to a ventilated thermocline. Below this the internal thermocline is diffusive transition region, and in the limit of small diffusivity it becomes an internal boundary layer between the ventilated thermocline and the abyss. The thermocline is, typically, baroclinically unstable and this leads to an upper ocean populated by vigorous mesoscale eddies. The eddies are strongest in regions of western boundary currents, 'mode water' regions, and in the circumpolar current, and in these regions the eddies significantly affect the structure of main thermocline. Elsewhere, the structure of the upper (ventilated) thermocline is largely determined by mean-flow advection. Lower in the water column, eddies typically tend to thicken the isostads that form the internal thermocline, leading to a complex three-way balance between mean flow, eddy fluxes and diffusion, suggesting that that the internal thermocline may have finite thickness even as diffusivity tends to zero. In the circumpolar current eddies are a dominant effect, and qualitatively change the structure of the stratification.

Key words: Dynamics, oceanography, thermocline, eddies, turbulence

1. Background

The modern development in our understanding of the thermocline is often considered to have begun with two back-to-back papers in 1959 in the journal *Tellus*. Welander (1959) suggested an adiabatic model, based on the ideal-fluid thermocline equations (i.e., the planetary geostrophic equations, with no diffusion terms in the buoyancy equation), whereas Robinson and Stommel (1959) proposed a model that is intrinsically diffusive. In the latter model [developed further by Stommel and Webster (1962)] the thermocline is diffusive front that forms at the convergence of two different homogeneous water types, warm near surface fluid and cold abyssal fluid below, whose thickness decreases as the diffusivity falls. The diffusive model was developed further by Salmon (1990) who found that numerical solutions of the planetary geostrophic equations can show a tendency to develop into a 'two-fluid' state — a pool of warm subtropical near surface water separated from a cold abyss by a diffusive front, an internal boundary layer, which Salmon associated with the main thermocline. Meanwhile, throughout the 1960s and 1970s the adiabatic model had continued its own development (see Veronis, 1969), culminating in the ventilated thermocline model of Luyten et al. (1983) (the 'LPS model') and its continuous extensions (Huang, 1988; Lionello and Pedlosky, 2000). Noting the clear difference between the two classes of theory, Welander (1971) commented that 'interior diffusive regions' may be necessary below an adiabatic upper-ocean thermocline. Samelson and Vallis (1997a) eventually suggested a model in which the upper thermocline is adiabatic, as in the ventilated thermocline model, but has a diffusive base that for small diffusivity constitutes an internal boundary layer. This model differs from the original diffusive models (e.g., the Robinson-Stommel-Webster model) and the model of Salmon in that the upper boundary conditions of the boundary layer are obtained (in principle) from matching the boundary layer to the lowest layer of a ventilated thermocline model. Thus, in this model, the thermocline has two dynamical regimes — an upper 'ventilated' region in which the dynamics are essentially adiabatic (or at least adiabatic below the mixed layer), and a lower diffusive layer. To the extent that the upper layer follows the dynamics of the LPS model of the thermocline, it will display features associated with that model — an eastern shadow zone and a western pool region for example. The relative strength of these two thermocline regimes depends on the geography of the situation: the upper advectively dominated thermocline is a mapping of the the horizontal meridional temperature gradient across the subtropical gyre; whereas the lower diffusively dominated thermocline is a mapping of the surface temperature across the subpolar gyre, and taken together they constitute the main thermocline.

The role of mesoscale eddies in all of this has only recently been investigated, and their role is now slowly emerging. That eddies may play a role in ocean circulation has, of course, long been conjectured and even accepted. For example Rhines and Young (1982) pointed out that, because potential vorticity is conserved on parcels save for the effect of mild diffusive processes, it (potential vorticity) will tend to become homogenized by the effect of eddies where the circulation both forms closed gyres and is shielded from the direct effect of surface forcing. Thus, we might expect regions of homogenized potential vorticity in, for example, the subsurface recirculation regions of the subtropical gyre. Because their theory is quasigeostrophic, it takes the vertical structure of the stratification as given and thus is not a theory of the thermocline *per se.* (However, it did anticipate the eastern shadow zone, as well as provide a mechanism for setting subsurface layers into motion that is different from that of the LPS model.) More recently, the possible role of eddies in actually setting ocean stratification has been explored. Vallis (2000b) noted that classical thermocline theories implicitly assume that mesoscale eddies are not important in setting the stratification, and the agreement or otherwise of such theories with observation will be one test of that assumption. Directly testing such theories against observation is of course very difficult, and the use of eddy resolving numerical models as an intermediary is likely to be a *sine qua non* of any such activity. Marshall et al. (2002) explicitly asked whether mesoscale eddies might play a role in setting the structure of the stratification in the upper ocean and suggested a simple model of that stratification, and Karsten et al. (2002) argued that mesoscale eddies set the stratification in the Antarctic Circumpolar Current (ACC). The stratification of the ACC differs from that of the subtropical gyres because one cannot build on Sverdrup balance to obtain a reasonable theory of the thermocline in the manner, say, of LPS. Because of the absence of meridional boundaries an E-W pressure gradient cannot be maintained over broad regions of the ACC and, by geostrophic balance, a mean meridional flow cannot be sustained. A consequence of this is that, in the absence of mesoscale eddies or their parameterized effects, the isopycnals become almost vertical and the convection is too deep (Vallis, 2000a). This is a highly baroclinically unstable situation, suggesting that if eddies are allowed to form they will have a first-order influence on the stratification. Recent numerical simulations do suggest that eddies may indeed play a role in determining the stratification and heat balance of the ocean, certainly in the ACC and, perhaps to a lessor degree (but still importantly), in the subtropical thermocline.

The rest of this paper expands on the above remarks to form a somewhat subjective review of some of the theoretical and numerical developments in thermocline dynamics. It is not a comprehensive review of all aspects of ocean stratification, or even of the theoretical aspects, and there is no discussion of the observations. Nevertheless, I hope readers will find it useful.

2. The Classical Picture

In this section we review the classical picture, by which we mean the picture described by the planetary geostrophic equations and that takes no account of mesoscale eddies. (Sometimes ‘classical’ is used in a complimentary way, meaning having stood the test of time, and sometimes in a derogatory way as a euphemism for ‘wrong.’ Our use here is neutral.)

2.1. EQUATIONS OF MOTION

The planetary geostrophic equations in the Boussinesq approximation are

$$\frac{\partial b}{\partial t} + \boldsymbol{v} \cdot \nabla b = \kappa \frac{\partial^2 b}{\partial z^2} \tag{1a}$$

$$-fv = -\phi_x \tag{1b}$$

$$fu = -\phi_y \tag{1c}$$

$$b = \phi_z \tag{1d}$$

$$\nabla \cdot \boldsymbol{v} = 0 \tag{1e}$$

The notation is standard, saline effects are omitted and a linear equation of state is implicit. Thus, the buoyancy b is given by $b = g\alpha\Delta T$, where α the coefficient of thermal expansion and ΔT is the temperature perturbation from a reference value, ϕ is pressure divided by a constant density, $\boldsymbol{v}$ is the three-dimensional velocity field and κ is a diffusivity. In oceanography these equations were originally known as the *thermocline equations* (Robinson and Stommel, 1959; Welander, 1959); a presentation from a more atmospheric perspective was given by Burger (1958), and Phillips (1963) and Pedlosky (1987) subsequently gave rather more systematic derivations. The diffusive term on the right-hand-side of the thermodynamic equation is generally regarded as representing a real physical process, namely turbulent diffusion at small scales. Even with this term the equations as written above are of insufficiently high order to be well-posed in a laterally closed domain, and small additional terms are needed in both the thermodynamic and momentum equations if the equations are to be solved in such a domain without numerical boundary layers (Samelson and Vallis, 1997b). The 'ideal' thermocline equations have no dissipative (diffusive or viscous) terms at all in the thermodynamic or momentum (geostrophic) equations, and thus cannot support the presence of lateral boundaries.

The planetary geostrophic equations are valid only for scales larger than the deformation radius and for scales for which the Coriolis parameter varies by an O(1) amount. The absence of nonlinear terms in the momentum equations means that baroclinic instability is incorrectly described: the equations have an ultraviolet catastrophe in that the growth rate increases monotonically with wavenumber (Colin-de-Verdiere, 1986). However, for scales of order the deformation radius and larger, baroclinic growth rates are *underestimated* (Smith and Vallis, 1998), and in fact is sufficiently small that it may readily be controlled with a modest amount of dissipation — smaller than might be required in a primitive equation model. The upshot of all this is with a grid resolution coarser than the deformation scale (say a grid scale of 1° or greater) steady solutions of the planetary geostrophic equations with small lateral and vertical diffusion are somewhat easier to obtain than corresponding solutions with the primitive equations, and unphysical cross-isopycnal diffusion (the 'Veronis effect') is diminished.

2.2. SCALING

2.2.1. *Advective Scaling*

A scaling for the depth of the thermocline can be obtained from Sverdrup balance in conjunction with the thermal wind equation. These equations are, respectively,

$$\beta v = f \frac{\partial w}{\partial z}, \tag{2}$$

with corresponding scaling

$$\beta V \sim \frac{f W_E}{D_a}, \tag{3}$$

and

$$f \frac{\partial v}{\partial z} = \frac{\partial b}{\partial x} \tag{4}$$

with corresponding scaling

$$\frac{f V}{D_a} \sim \frac{\Delta b}{L}. \tag{5}$$

In these equations Δb is the magnitude of the buoyancy variation and the other scaling variables are denoted with a capital letter. Thus, L is the horizontal scale of the motion, which we take as the gyre or basin scale, and the parameter D_a is the vertical scale where the subscript a denotes an advective scale. The appropriate vertical velocity to use is that due to Ekman pumping, W_E; we will assume *a priori* that this is much larger than the abyssal upwelling velocity, which in any case is zero by assumption at $z = -D_a$. W_E and L are thus given by the geometry and the strength of the wind forcing, whereas U and D_a are part of the solution. Eliminating V from (3) and (5) gives the estimate

$$D_a \sim \left(\frac{W_E f^2 L}{\beta \Delta b} \right)^{1/2} \tag{6}$$

which has its roots in Welander (1959). We can also derive this result by imagining the upper ocean to be a single layer of homogeneous fluid of variable depth h lying over a resting abyss, then (4) can be replaced by $f v = g' \partial h / \partial x$ where g' is the usual reduced gravity and h is, effectively, the depth of the thermocline. This equation scales like

$$f V \sim g' \frac{D_a}{L} \tag{7}$$

and using this and (3) we obtain $D_a \sim \left(f^2 L W_E / \beta g' \right)^{1/2}$, which is equivalent to (6), because $\Delta b \sim g'$ is just the reduced gravity across the thermocline.

As an aside, we note that if the horizontal scale is large enough so that there is little cancellation in the terms comprising horizontal divergence, then the mass conservation equation, $\partial u/\partial x + \partial v/\partial y = -\partial w/\partial z$, scales like $V/L \sim W/D_a$. Using this with (5) gives

$$D_a \sim \left(\frac{W_E f L^2}{\Delta b}\right)^{1/2}. \tag{8}$$

which is the same as (6) if $\beta \sim f/L$, that is for the planetary scale.

Thus, the depth of the wind-influenced region is proportional to the half-power of the strength of the wind-stress, and inversely proportional to half power of the meridional temperature gradient. The former dependence is reasonably intuitive, the latter perhaps less so. One way to think about this is that as the temperature gradient increases, the associated thermal wind-shear V/D_a correspondingly increases. But if the mechanical forcing is unaltered, then Sverdrup balance can only remain satisfied if the vertical scale of the motion decreases. From a shallow water perspective, that interface displacements tend to fall as the reduced gravity increases is a familiar notion, but the constraining effects of Sverdrup balance are such that the thermocline depth does not fall as rapidly as g' increases (equations (3) and (7)).

Finally, let us estimate the thermocline depth as given by such scalings. Using $W_E \sim 10^{-6}\,\mathrm{m\ s^{-1}}$, $\Delta b = g\Delta\rho/\rho_0 = g\alpha\Delta T \sim 10^{-2}\,\mathrm{m\ s^{-2}}$, $L = 5000$ km and $f = 10^{-4}\ \mathrm{s^{-1}}$ in (6)

$$D \sim 500\ \mathrm{m} \tag{9}$$

However, this is only an estimate, and might easily be in error by some nondimensional number not revealed by the scaling analysis. One hopes any such numbers are $\mathcal{O}(1)$.

2.2.2. *Diffusive Scaling*

In the derivation above the diffusion in the thermodynamic equation plays no role; indeed the thermodynamic equation provides no additional vertical scale. The vertical velocity is imposed by the Ekman layer, and a closed scaling is obtained from the mass conservation and Sverdrup balance equations alone. Deeper in the thermocline, the vertical velocity will diminish and no longer be dominated by the Ekman pumping; it will be affected by upwelling from the abyss and should be part of the solution provided by the scaling rather than an imposed parameter. The steady thermodynamic equation, with diffusion,

$$\boldsymbol{v} \cdot \nabla b = \kappa \frac{\partial^2 b}{\partial z^2} \tag{10}$$

implies the scales

$$\frac{U}{L},\ \frac{W}{\delta} \sim \frac{\kappa}{\delta^2}. \tag{11}$$

where δ is a vertical scale. This scaling must be consistent with the Sverdrup relation, (2), and thermal wind, (4), now with scalings

$$\beta V \sim \frac{fW}{\delta}, \tag{12}$$

and

$$\frac{fV}{\delta} \sim \frac{\Delta b}{L}. \tag{13}$$

Assuming that $\beta/f \leq 1/L$ then these relations give the diffusive vertical scale

$$\delta \sim \kappa^{1/3}\left(\frac{fL^2}{\Delta b}\right)^{1/3} \tag{14}$$

and the internal vertical velocity scale

$$W \sim \frac{\kappa}{\delta} \propto \kappa^{2/3} \tag{15}$$

which is a scaling for the strength of the overturning circulation. If the vertical advection term dominates in (11) then we can obtain the scaling

$$\delta \sim \kappa^{1/3}\left(\frac{f^2 L}{\beta \Delta b}\right)^{1/3}. \tag{16}$$

but for most purposes this is not qualitatively different.

Using $f = 10^{-4}\,\mathrm{s}^{-1}$, $L = 5\,10^{-6}\,\mathrm{m}$, $g = 10\ \mathrm{m\,s}^{-2}$, $\kappa = 10^{-5}\,\mathrm{m}^2/\mathrm{s} = 0.1\ \mathrm{cm}^2/\mathrm{s}$, $\Delta b = g\Delta\rho/\rho_0 = g\alpha\Delta T$ and $\Delta T = 10\,K$ gives $\delta \sim 130$ m. These parameter values also give $U \sim 0.3\ \mathrm{cm/s}$ and $W \sim 10^{-5}\,\mathrm{cm/s}$. Note that W_E (which is typically about $10^{-4}\,\mathrm{cm/s}$) is indeed much larger than W.

The scaling above assumes that the length scale over which thermal-wind balance holds is the gyre scale itself. In fact there is another length scale that is more appropriate, namely the horizontal difference across the sloping thermocline, and this leads to a slightly different scaling for the thickness of the thermocline. The depth of the subtropical thermocline is not constant; rather it slopes (see fig. 1). It shoals up to the east simply because of Sverdrup balance, and it may slope up polewards because the curl of the windstress falls (and is zero at the polewards edge of the subtropical gyre). Thus, the appropriate horizontal length scale $\hat{L}$ is not the basin scale itself, but is given by

$$\hat{L} = \delta\frac{L}{D}. \tag{17}$$

This is no longer an external imposed parameter, but must be determined as part of the solution. Using $\hat{L}$ instead of L as the length scale gives, after just a little algebra, the modified diffusive scale

$$\delta_w = \kappa^{1/2}\left(\frac{fL^2}{\Delta b D_a}\right)^{1/2} = \kappa^{1/2}\left(\frac{fL^2}{\Delta b W_E}\right)^{1/4}. \tag{18}$$

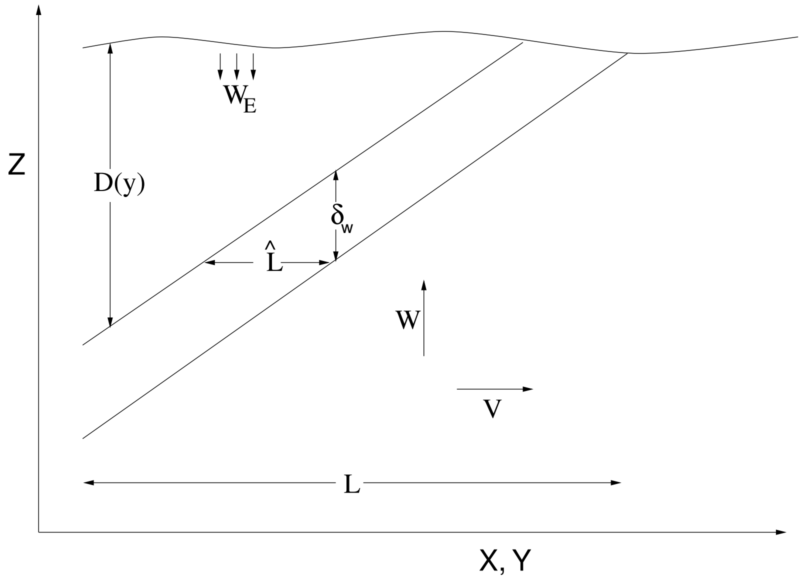

Figure 1. Scaling for the thermocline. If the thermocline slopes, the appropriate horizontal scale is given by $\hat{L}$, which is part of the solution rather than being imposed.

Substituting values of the various parameters results in a thickness of about 100–200 m, for both (14) and (18). This is somewhat less that the scaling estimate (6) of the depth of the advective, upper thermocline. Observations, arguably, do not suggest that this is the case. This might be because some other process is thickening the lower thermocline or, perhaps more likely, because all of these scalings are likely to be in error by a nondimensional factor which will differ from case to case.

Note that (18) suggests that the thermocline depth scales as $\kappa^{1/2}$. This scaling is the appropriate one for a wind- and buoyancy-driven ocean, whereas (14) is appropriate if there is no wind forcing, and this expectation is borne out by numerical simulations Vallis (2000a). The vertical velocity, and hence the meridional overturning circulation, now scales as

$$W \sim \frac{\kappa}{\delta_w} \propto \kappa^{1/2} \tag{19}$$

rather than $\kappa^{2/3}$.

2.3. THE LOWER THERMOCLINE AS A BOUNDARY LAYER

If the lower thermocline is indeed a boundary layer, then it is natural to try to apply some of the techniques of boundary layer theory. To do this we first combine the planetary geostrophic equations into a single equation, the ‘M-equation’. (This section may be skipped at first reading.)

2.3.1. *The M–equation*

The planetary geostrophic equations can be written as a single partial differential equation in a single variable, although the resulting equation is of quite high order (third, in the absence of friction) and nonlinear. Cross differentiating the geostrophic relations, and using mass continuity, implies the Sverdrup relation

$$\beta v = f \frac{\partial w}{\partial z} \tag{20}$$

or, using geostrophic balance again

$$\frac{\partial \phi}{\partial x} + \frac{\partial}{\partial z}\left(-\frac{f^2}{\beta} w\right) = 0. \tag{21}$$

This equation is automatically satisfied if

$$\phi = M_z \quad \text{and} \quad \frac{f^2 w}{\beta} = M_x. \tag{22}$$

Then straightforwardly

$$u = -\frac{\phi_y}{f} = -\frac{M_{zy}}{f} \qquad \text{and} \qquad v = \frac{\phi_x}{f} = \frac{M_{zx}}{f}, \tag{23}$$

and

$$b = M_{zz}. \tag{24}$$

The thermodynamic equation becomes

$$\frac{\partial M_{zz}}{\partial t} + \left[\frac{-M_{zy}}{f} M_{zzx} + \frac{M_{zx}}{f} M_{zzy}\right] + \frac{\beta}{f^2} M_x M_{zzz} = \kappa M_{zzzz} \tag{25}$$

or

$$\frac{\partial M_{zz}}{\partial t} + \frac{1}{f} J(M_z, M_{zz}) + \frac{\beta}{f^2} M_x M_{zzz} = \kappa M_{zzzz}. \tag{26}$$

This is the ‘M–equation,’ first derived by Welander (1959). It is analogous to the potential vorticity equation in quasi-geostrophic theory in that it expresses the entire dynamics of the system in a single, nonlinear, advective-diffusive partial differential equation. The Sverdrup relation in the interior is automatically satisfied, and at the surface it is represented by

$$\frac{\beta}{f^2} M_x(x, y, z = 0) = w_E. \tag{27}$$

Equation (26) is rather complicated; analytic solutions are very hard to find, and numerically it is easier to find solutions by integrating the original planetary geostrophic equations. However, it is possible to move forward by appropriately approximating the equation, and the next two sections discuss this.

2.3.2. *A simple one-dimensional model*

A simple special case illustrates clearly the formation of an internal front. If $M = M(y, z)$ then the M-equation becomes

$$\frac{\beta}{f^2} M_x M_{zzz} = \kappa M_{zzzz} \tag{28}$$

which represents the advective-diffusive balance $wb_z = \kappa b_{zz}$. The horizontal advection terms vanish because the zonal velocity (u) and the meridional temperature gradient (b_y) are each zero. We consider, following Salmon (1990), the special case

$$M = xW(z) \tag{29}$$

whence (28) becomes

$$WW_{zzz} = \kappa W_{zzzz}. \tag{30}$$

A closely related equation was put forward by Stommel and Webster (1962) and Young and Ierley (1986), namely

$$\left[2W - zW_z\right] W_{zzz} = \kappa W_{zzzz}. \tag{31}$$

where the vertical velocity is proportional to the term in square brackets. The similarity of the time-dependent form of these equations (e.g. $W_{zzt} + WW_{zzz} = \kappa W_{zzzz}$) to Burger's equation ($W_t + WW_z = \kappa W_{zz}$) suggests that fronts might form. Appropriate boundary conditions for (30) are a prescribed buoyancy or buoyancy flux and vertical velocity at the upper and lower boundaries, for example

$$\begin{aligned} W &= W_E, \qquad b_z = W_{zzz} = 0 \qquad \text{at top} \\ W &= 0, \qquad b_z = W_{zzz} = 0 \qquad \text{at bottom} \end{aligned} \tag{32}$$

where W_E is the vertical velocity at the base of the top Ekman layer, which is negative for Ekman pumping in the subtropical gyre. We obtain solutions numerically by Newton's method; both (30) and (31) exhibit qualitatively similar solutions, and a typical one is shown in fig. 2.

The important point is that the solutions to (30) and (31) generically produce an interior front, whose thickness goes to zero as $\kappa \to 0$. These equations are rational simplifications of the full three dimensional equations of motion, suggesting that these equations, too, may display frontal behaviour in the limit of small diffusivity. This front occurs where the vertical velocity is zero — where the downwards Ekman pumping and the upwelling from the abyss 'collide', and the water mass properties change rapidly in the vertical.

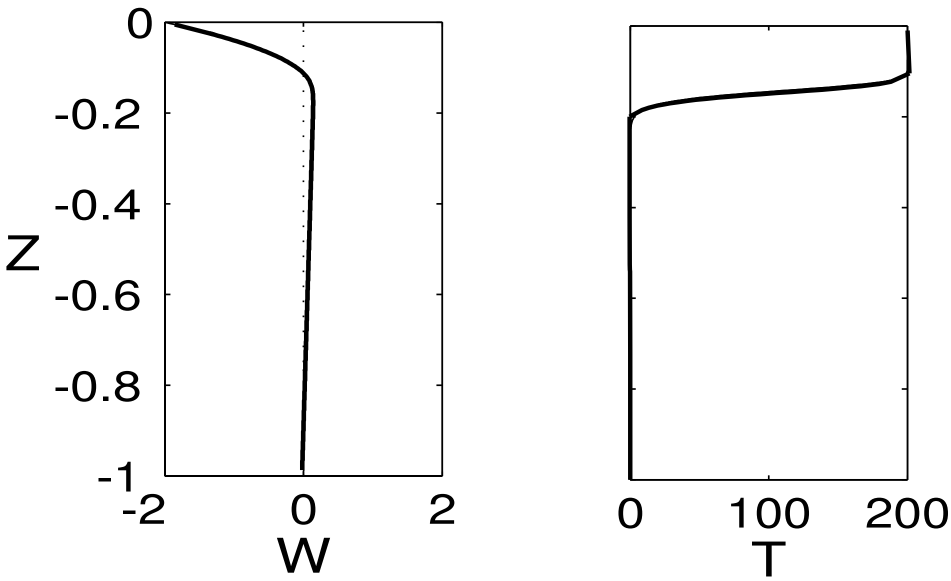

Figure 2. A numerical solution of (31), obtained using Newton's method. Buoyancy flux is zero at top and bottom, and there is an imposed downward vertical velocity at the top. The solution exhibits weak upwelling in the lower part of the domain, and a *front* forms where the vertical velocity passes through zero.

2.3.3. *Boundary layer analysis*

Following Samelson (1999) let us *assume* that the temperature varies rapidly only in an 'internal boundary layer' of thickness δ; above and below this it is assumed to be at most slowly varying. Thus we write

$$b(x, y, z) = b(x, y, z) + \hat{b}(x, y, \zeta) \tag{33}$$

where $\hat{b}$ is important only in the boundary layer and ζ is a stretched co-ordinate such that

$$z + h(x, y) = \delta\zeta. \tag{34}$$

That is, it is the vertical distance from $z = -h$, scaled by the boundary layer thickness δ, and is presumptively an $O(1)$ quantity. Note that we allow the depth of the boundary layer to change as a function of horizontal co-ordinate. Then M varies as

$$M = M(x, y, z) + \delta^2 \hat{M}(x, y, \zeta) \tag{35}$$

where $\hat{M} \to 0$ away from the boundary layer. The scaling factor on $\hat{M}$ is needed because $b = M_{zz}$, and so $b \sim (1/\delta^2)M$ in the boundary layer. Since b remains an order one quantity throughout, $\hat{M}$ must be scaled appropriately.

In the boundary layer the derivatives of M become

$$\frac{\partial \hat{M}}{\partial z} = \frac{1}{\delta}\frac{\partial \hat{M}}{\partial \zeta} \tag{36}$$

and

$$\begin{aligned}\frac{\partial \hat{M}}{\partial x} &= \frac{\partial \hat{M}}{\partial \zeta}\frac{\partial \zeta}{\partial x} + \frac{\partial \hat{M}}{\partial x} \\ &= \frac{\partial \hat{M}}{\partial \zeta}\left(\frac{1}{\zeta}\frac{\partial h}{\partial x}\right) + \frac{\partial \hat{M}}{\partial x}\end{aligned} \tag{37}$$

Substituting these into (25) we obtain, for a steady state,

$$\begin{aligned}&\delta\left\{\frac{1}{f}\left(\hat{M}_{\zeta x}\hat{M}_{\zeta\zeta y} - \hat{M}_{\zeta y}\hat{M}_{\zeta\zeta x}\right) + \frac{\beta}{f^2}\hat{M}_x\hat{M}_{\zeta\zeta\zeta}\right\} + \frac{\beta}{f^2}h_x\hat{M}_\zeta\hat{M}_{\zeta\zeta\zeta} \\ &+\frac{1}{f}\left[h_x\left(\hat{M}_{\zeta\zeta}\hat{M}_{\zeta\zeta y} - \hat{M}_{\zeta y}\hat{M}_{\zeta\zeta\zeta}\right) + h_y\left(\hat{M}_{\zeta x}\hat{M}_{\zeta\zeta\zeta} - \hat{M}_{\zeta\zeta}\hat{M}_{\zeta\zeta x}\right)\right] \\ &\qquad = \frac{1}{\delta^2}\kappa\hat{M}_{\zeta\zeta\zeta\zeta}.\end{aligned} \tag{38}$$

(The horizontal advective terms of order δ^{-1} vanish identically.) Obviously, this equation of itself does not provide much insight even to the most algebraically minded oceanographer. But it is, nevertheless, revealing of the its scaling behavior.

If $h_x = h_y = 0$ (i.e., the base of the thermocline is flat), (38) becomes

$$\frac{1}{f}\left[\hat{M}_{\zeta x}\hat{M}_{\zeta\zeta y} - \hat{M}_{\zeta y}\hat{M}_{\zeta\zeta x}\right] + \frac{\beta}{f^2}\hat{M}_x\hat{M}_{\zeta\zeta\zeta} = \frac{1}{\delta^3}\kappa M_{\zeta\zeta\zeta\zeta}. \tag{39}$$

Since all the terms in this equation are, by construction, order one, we immediately see that

$$\delta \propto \kappa^{1/3}, \tag{40}$$

just as in the scaling arguments. On the other hand, if h_x and/or h_y are order one quantities then the dominant balance in (38) is

$$\frac{1}{f}\left[h_x(\hat{M}_{\zeta\zeta}\hat{M}_{\zeta\zeta y} - \hat{M}_{\zeta y}\hat{M}_{\zeta\zeta\zeta}) + h_y(\hat{M}_{\zeta x}\hat{M}_{\zeta\zeta\zeta} - \hat{M}_{\zeta\zeta}\hat{M}_{\zeta\zeta x})\right] = \frac{1}{\delta^2}\kappa M_{\zeta\zeta\zeta\zeta} \tag{41}$$

and

$$\delta \propto \kappa^{1/2}, \tag{42}$$

which again is consistent with the scaling arguments. Thus, if the isotherm slopes are fixed independently of κ (perhaps by the windstress), then as $\kappa \to 0$ an internal boundary layer will form whose thickness is proportional to $\kappa^{1/2}$. We expect this to occur be at the base of the main thermocline, with purely advective dynamics being dominant in upper part of the thermocline, and determining the slope of the isotherms (i.e., the form of h_x and h_y), as in fig. 1.

Interestingly, the balance in the boundary layer equation does not simply correspond to $wb_z \approx \kappa b_{zz}$. Both at $\mathcal{O}(1)$ and $\mathcal{O}(\delta)$ the horizontal advective terms in (38) are of the same asymptotic size as the vertical advection terms. In the middle of the internal boundary layer the thermodynamic balance is thus $\boldsymbol{u} \cdot \nabla_h b + wb_z \approx \kappa b_{zz}$, with all terms in principal important. We might have anticipated this, because the vertical velocity, and the second derivative of buoyancy, passes through zero within the boundary layer. [More discussion is given in Samelson (1999).] Finally, we note that even though the balance at the thermocline base *necessarily* involves diffusion, diffusion (if small) plays a small role in the thermocline heat budget. It is then *not* a significant mechanism for transporting heat or the formation of water masses in the fluid interior; it is not the case for example that a net input of buoyancy at the surface over the subtropical gyre is balanced by an interior diffusive flux across isothermal surfaces. Diffusion is, however, necessary for the maintenance of a meridional overturning circulation, via (15) or (19).

2.4. SUMMARY

The classical line of investigation might be summarized in fig. 3, with a ventilated thermocline lying on top of an internal thermocline. The former describes the structure of isopycnals that outcrop in the subtropical gyre where Ekman pumping is downwards; this merges smoothly into the internal thermocline, which spans those isopycnals that outcrop in the subpolar gyre. Perhaps the most serious shortcoming of this as a conceptual model (rather than a quantitative one, which would need realistic geometry, salinity, etc) is that no account is taken of mesoscale eddies, and we now turn our attention to this matter.

3. Geostrophic Turbulence in the Thermocline

The main thermocline is a region of horizontal temperature gradients and vertical shear; thus, we might expect it to be a region of baroclinic instability — as pointed out by Gill et al. (1974), Robinson and McWilliams (1974) and others. Since the time of these papers both observations and simulations with eddying General Circulation Models (e.g., Smith et al., 2000) have indicated the ubiquity of eddies in the midocean and, especially, in the regions of the western boundary currents and their immediate extensions. Furthermore, the scale separation between the first deformation radius ($\sim$ 100 km) and the gyre scale ($\sim$ 1000 km or more) suggests that notions of geostrophic turbulence might be even more applicable in the ocean than in the atmosphere where the scale separation is much weaker. The simplest picture (Rhines, 1977; Salmon, 1980) assumes a uniformly stratified ocean in

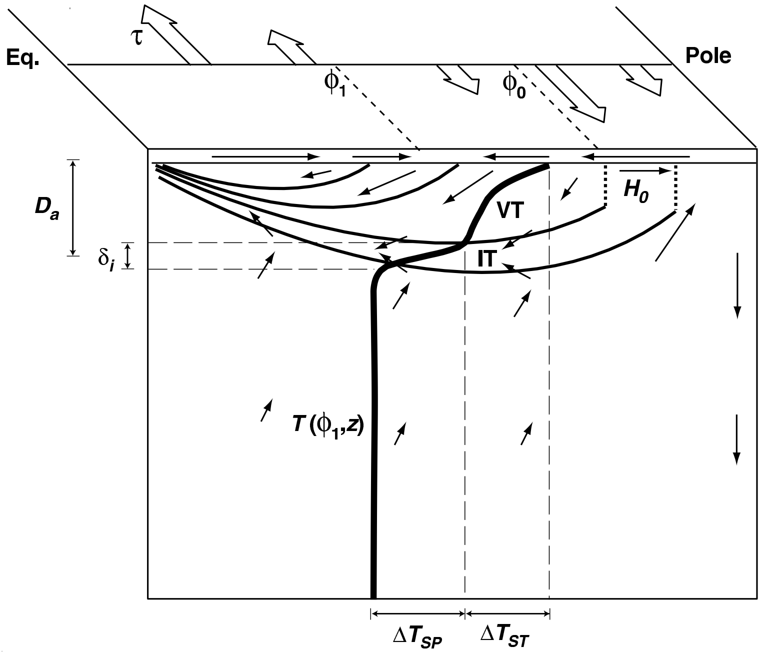

Figure 3. A schematic of the structure of the classical wind- and buoyancy-driven thermocline, according to SV. The medium-weight lines are isopycnals, 'IT' denotes the advective-diffusive internal thermocline, 'VT' denotes the advective ventilated thermocline, and the thick solid line is a temperature profile through the middle of the subtropical gyre. The single arrows represent the meridional circulation, and the double arrows the overlying windstress. Buoyancy forcing may be considered to be low-latitude surface warming and high-latitude surface cooling.

which a mean shear is maintained at very large scales. Baroclinic instability then leads to the growth of both baroclinic and barotropic modes at scales near the first radius of deformation, followed by an inverse cascade of barotropic energy back to large scales. Dissipative process are necessary to remove energy in the inverse cascade and enstrophy at small scales.

An interesting variation on this model arises when the stratification is nonuniform, as in the subtropical thermocline. The dominant effect is to inhibit the inverse cascade of energy in the barotropic mode. This is illustrated most clearly in an initial value problem using the quasi-geostrophic equations in a doubly-periodic domain (Smith and Vallis, 2001). Two experiments were carried out differing only in their stratification: in one configuration the stratification N^2 is uniform, whereas in the other the stratification is enhanced near the surface to represent the thermocline. The initial energy is all in the baroclinic modes and confined to very large horizontal scales, representing the energy in the very large scale flow. In the experiment with uniform stratification, the flow of energy largely follows the picture above, with a nonlocal transfer of energy to both baroclinic and barotropic modes at the deformation radius (this is just the usual 'baroclinic instability') followed by an inverse cascade of barotropic energy to larger scales (fig. 4). In the parlance of baroclinic lifecycles, the process is one of 'baroclinic growth and barotropic decay.' It is noteworthy that friction is not a necessary ingredient to this cycle: the cycle still decays barotropically and although energy may pile up at large barotropic scales this does not lead to another round of instability.

If the stratification is thermocline-like, then the transfer first proceeds first to the baroclinic mode, and only subsequently to the barotropic mode, where it may then be transferred to larger scales (fig. 5). That is, the transfer to the barotropic mode is inhibited, as if there is a constriction in the pipeline of the baroclinic lifecycle. In statistically steady experiments, this leads to a slightly enhanced level of activity in the baroclinic mode at the scale of the deformation radius. This, coupled with fact that the ocean is not uniformly subject to baroclinic instability, and therefore that baroclinic eddies may not always find themselves surrounded by other eddies with which to cluster and form an inverse cascade, may be the reason that the baroclinic eddy scales in the ocean are comparable with the first deformation radius. However, we might still expect that the barotropic energy is at a larger scale, perhaps determined by the Rhines scale or frictional processes.

4. Effect of Eddies on the Structure of the Thermocline

We come, finally, to the issue as to whether and how mesoscale eddies substantially affect the structure of the steady of the thermocline and whether and how classical models, such as those of the previous sections, need to be modified. At the time of writing (2003) this is an open question and, although recent numerical simulations and various theoretical models do suggest that eddies play a role in setting thermocline structure, the picture is not wholly clear.

4.1. CRUDE *A PRIORI* SCALING

Consider first the advectively dominated upper thermocline. An estimate of the importance of mesoscale eddies arises may be roughly estimated by comparing the sizes of the two terms

$$\nabla \cdot \overline{\boldsymbol{u}'b'} \qquad \text{and} \qquad \nabla cdot(\overline{\boldsymbol{u}}\overline{b}). \tag{43}$$

If we take $b' \sim \delta \boldsymbol{x} \cdot \nabla \overline{b} \sim L' \Delta b / L$, where Δb and L refer to the scales of mean quantities, then the ratio of the magnitude of the mean advection to that of the eddy advection is characterized by an eddy Peclet number

$$Pe = \frac{UL}{U'L'} \tag{44}$$

where U' is the eddy velocity scale, U is the velocity scale of the mean flow, and L' and L are the length scales of the eddies and mean flow, respectively. The denominator is just an estimate of the size of an eddy diffusivity.

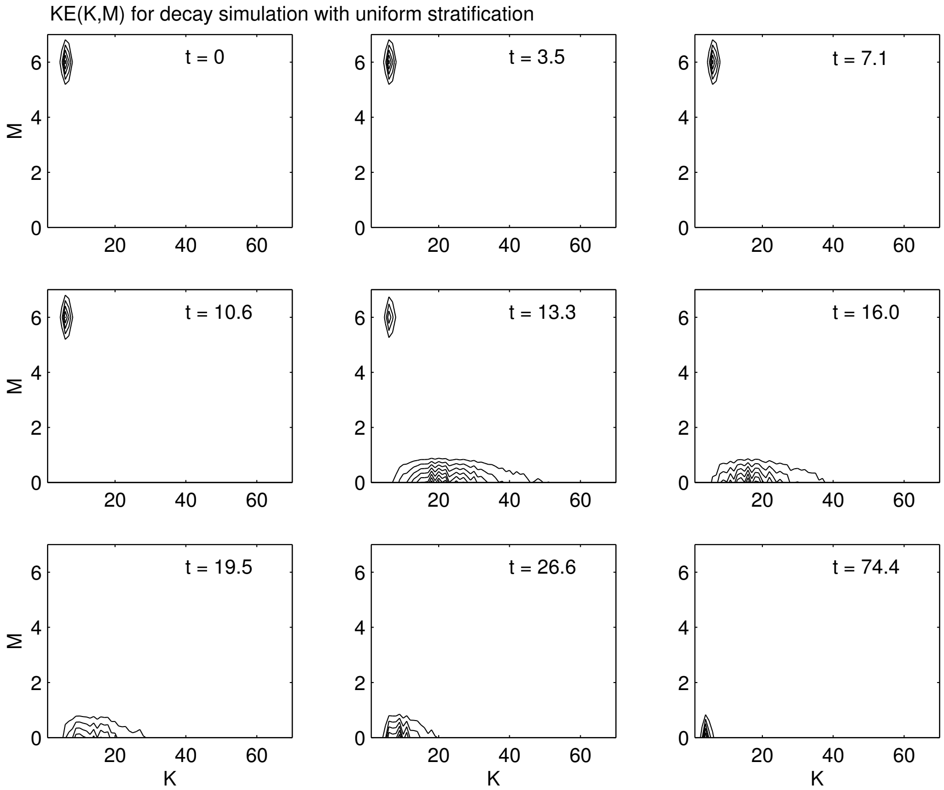

Figure 4. Time sequence of kinetic energy spectra for in a nearly inviscid stratified quasi-geostrophic simulation with uniform stratification. Times are given in terms of eddy turn-around time, τ_{eddy}, and axes are vertical mode number, M, and horizontal isotropic wavenumber, K. Contour values are linear over the range of values at each frame. The first radius of deformation is at wavenumber 15. From Smith and Vallis (1998).

We can estimate the ratio L'/L if we take L' to be the deformation radius and if we assume, initially, that eddies have only a perturbative effect on the depth of the thermocline. The deformation radius is

$$L_d \sim \frac{ND}{f} \tag{45}$$

where D is the depth of the thermocline and N is the Brunt-Väisälä frequency. An estimate for this is $(\delta b/D)^{1/2}$ where D is the depth of the thermocline and $\Delta b \sim g\alpha\Delta T$ is the change in buoyancy across it. If D is given by (6) then we find

$$L_d \sim \left(\frac{W_E \Delta b L}{\beta f^2}\right)^{1/4} \sim \left(\frac{W_E \Delta b L^2}{f^3}\right)^{1/4} \tag{46}$$

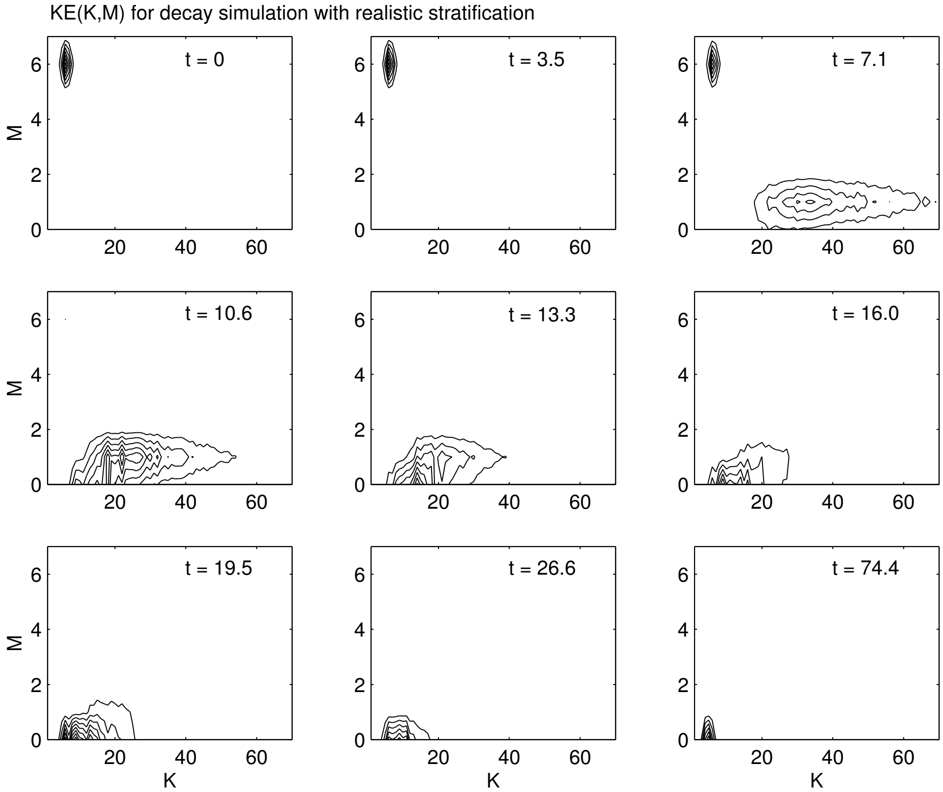

Figure 5. Time sequence of kinetic energy spectra for in a nearly inviscid stratified quasi-geostrophic simulation with enhanced stratification in the upper ocean, representing a thermocline. Times are given in terms of eddy turn-around time, τ_{eddy}, and axes are vertical mode number, M, and horizontal isotropic wavenumber, K. Contour values are linear over the range of values at each frame. The first radius of deformation is at wavenumber 24. From Smith and Vallis (1998).

where the second term holds for the planetary scale $\beta \sim f/L$. These are *a priori* estimates for the oceanic deformation scale. With the oceanically reasonable values of $g = 10\,\mathrm{m\,s^{-1}}$, $f = 10^{-4}\,\mathrm{s^{-1}}$, $\alpha = 10^{-4}\,\mathrm{K^{-1}}$, $L = 10^6\,\mathrm{m}$ and $\Delta T = 20$ K we find $L' \approx 50\,\mathrm{km}$ and $L'/L \approx 0.05$, with still smaller values for a (perhaps realistically) larger value of L. Observations suggest a similar value for the value of the deformation radius in the midlatitude ocean. Chelton et al. (1998), for example, find that the first deformation radius various between about 20 km and 100 km in midlatitudes, with values of about 50 km being typical in the subtropical gyres. Eddies themselves tend to be rather larger than this, perhaps reflecting not just a (rather weak) inverse cascade (section 3) but also the tendency of the length scale of maximum instability to be larger than the deformation radius. In the Eady problem, for example, the length-scale of maximum instability is about four times larger than the deformation radius.

Reliable theoretical estimates of the ratio of the eddy to the mean velocity are harder to come by. The simplest assumption of all is to suppose that $U' \sim U$ (Green, 1970; Stone, 1972), although this does not take into proper account the turbulent properties of the flow: for example, the eddy velocity will increase as the extent of any inverse cascade increases. Held and Larichev (1996) do incorporate these properties using the machinery of geostrophic turbulence, but their arguments are not likely to be quantitatively valid in the ocean, because of vertical inhomogeneities in the stratification and horizontal inhomogeneities in the eddy field itself. Except perhaps in the Antarctic Circumpolar Current, it seems unlikely that a fully developed geostrophically turbulent inverse cascade exists in the ocean; in subtropical gyres, for example, eddies will develop in or near the strongly baroclinically unstable western boundary current or its extension (e.g., the Gulf Stream extension, the Kuroshio extension) but then may pass into less eddy rich regions. Eddies then find it more difficult to interact with each other, and the inverse cascade is inhibited.

Observations suggest that the ratio of U'/U ranges from about 1 to 5 in the subtropical gyre, corresponding to ratios of eddy-to-mean kinetic energy from 1 to 30 (e.g., Stammer, 1997). If L/L' is about 5 or smaller, then the eddy Peclet number may drop toward unity or smaller, indicating the importance of eddies. The conclusion of all this is that the eddy Peclet number is probably larger than unity over most of the subtropical gyre (and therefore mean effects locally dominate), but approaches unity in eddy rich regions, such as mode water regions near the western boundary layer. This means we cannot *a priori* eliminate the possibility that eddies play a significant role in setting the structure of the thermocline, but we should also expect (or perhaps at least hope) that the structure of the non-eddying models remains relevant in eddying models, and in the real ocean.

Evidently, the importance of eddies to the heat and momentum budgets of the ocean depends on their size — if the eddies are as large as the mean flow then their effects will be correspondingly large. Green (1970) suggested that the eddy mixing length should be the size of the baroclinic zone, rather than the eddy size, and this was used by Marshall et al. (2002) in their arguments. If we additionally take $U' \sim U$, then the eddy Peclet number given by (44) is obviously $\mathcal{O}(1)$ and with this scaling eddies play a first order role in thermocline dynamics, and in ocean dynamics in general. Marshall et al considered the thermodynamic balance of a warm lens created by Ekman pumping and a surface buoyancy flux. In this case, the incoming buoyancy flux must be balanced by the integrated outgoing eddy fluxes over the lens. Then, further assuming that

$$\overline{v'b'} \sim U \Delta b_{\text{lens}} \tag{47}$$

and using also the thermal wind relation (in cylindrical coordinates)

$$f \frac{\partial u}{\partial z} = \frac{\partial \Delta \Delta b}{\partial r}, \tag{48}$$

they obtained a scaling for the depth of the lens

$$h_{\text{lens}} \sim \left(\frac{f}{B}\right)^{1/2} W_E L, \tag{49}$$

where L is the size (e.g., radius) of the lens and B is the surface buoyancy flux. This is actually the same as the classical scaling (8) if we take $B \sim W_E \Delta b$. It is not surprising we obtain the same result, because we have used the same parameters in the dimensional analysis, but the assumptions underlying the two scalings are quite different. This scaling is supported by laboratory and numerical results, although this is not perhaps a strong argument that the mixing scale in the real ocean is the scale of the baroclinic zone. That would only be the case if the eddy scale itself were the scale of the baroclinic zone, or if eddies were somehow organized on that scale to transfer properties efficiently across the baroclinic zone.

Regarding the lower, diffusive, thermocline one may ask whether an internal boundary layer forms even in the presence of vigorous eddies. Scaling arguments themselves are not immediately elucidating on this matter. For example, we might compare the sizes of

$$\nabla \cdot \overline{\boldsymbol{u}'b'} \qquad \text{and} \qquad \kappa \frac{\partial^2 b}{\partial z^2} \tag{50}$$

The ratio of these terms is approximately

$$\frac{U'L'\delta^2}{\kappa L^2} \tag{51}$$

where δ is a vertical scale. If the eddies have only a perturbative effect on the thickness of the thermocline, then we might use the vertical scale (14) or (18), but the resulting estimate is not very informative (although both scalings suggest a rather weak dependence on the value of the diffusivity itself, and no dependence at all if (18) is used). We might, nevertheless, expect eddy effects to be *more* important in the lower thermocline than the upper part, at least away from the surface (in the near surface region, eddies have an important diabatic effect, as we see later.) This may seem counterintuitive since eddies tend to be strongest near the surface; however, over their lifecycle eddies will, even if somewhat inefficiently, tend to barotropize or at least put energy into the first vertical mode, so distributing eddy energy over the entire depth of the thermocline. The intensity of the mean flow, however, falls off fairly rapidly with depth, so that the eddy Peclet number may similarly fall with depth, bringing with it the possibility of a subsurface balance involving eddy, mean and diffusive terms. The precise balance achieved becomes

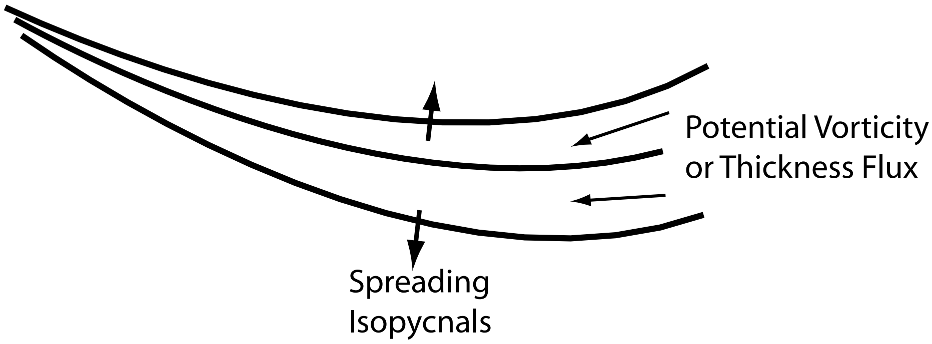

Figure 6. Schematic of the possible thickening of the internal thermocline by eddy effects. If mesoscale eddies tend to diffuse potential vorticity along isopycnals (solid lines) then if β is small they will tend to transport thickness from a region with greater separation between isopycnals to a region with less (in so far as is consistent with a loss of available potential energy), spreading the isopycnals and thickening the internal boundary layer.

a quantitative issue which probably cannot be addressed by armchair reasoning, and both observations and high resolution numerical simulations will be needed to definitively settle the issue.

In spite of this quantitative uncertainty, if eddies are even moderately active in the internal thermocline then they are likely to lessen the impact of diffusion in setting its structure. To see this, consider a model ocean in which diffusion is small, and in which eddies are not present. As in section 2 this leads to a thin diffusive internal thermocline. If eddies are now allowed to form, and if their leading order effect is to diffuse potential vorticity (or, approximately, thickness) along isopycnals, then the internal boundary layer will generically thicken, as illustrated in fig. 6. The thickness flux is subducted from the ocean mixed layer where thickness is 'generated' by diabatic ocean-atmosphere interactions. Eddies almost certainly play a role in this, but the important point is that there can be subduction of water mass into the internal thermocline. Because the original thickness in the noneddying case was such as to balance diffusion with mean advection, then in a thicker thermocline the diffusive term (proportional to $\partial^2 b/\partial z^2$) becomes correspondingly less important, and an internal boundary layer need not exist in the sense that the thickness of the internal thermocline remains finite as diffusivity tends to zero. Of course, the eddies might affect the mean flow in such a manner that diffusion remains important and thus, again, the argument is heuristic.

4.2. NUMERICAL RESULTS

We will now briefly describe some numerical calculations of an idealized but eddying ocean. We shall be rather descriptive here — for more detail, see Henning and Vallis (2003). [Radko and Marshall (2003) have also explored the influence of eddies on the thermocline in idealized numerical experiments.] The simulations

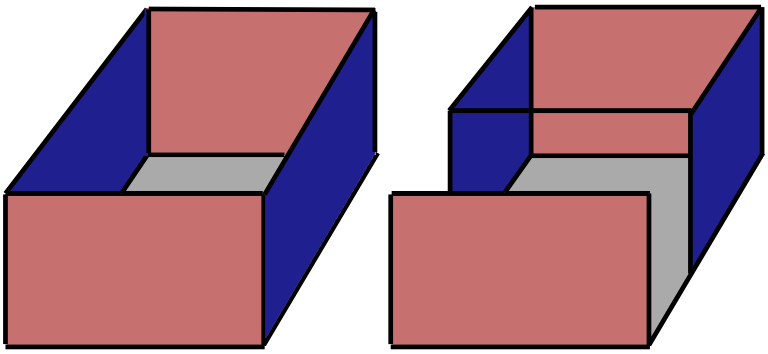

Figure 7. Domains used in primitive equation numerical experiments. Left: A box domain, with wind and buoyancy forcing are such as to give a subtropical gyre and a smaller subpolar gyre. Right: a re-entrant channel replaces the subpolar gyre, providing a simple model of the Antarctic Circumpolar Current.

were carried out in a smaller and more idealized domain and at a rather lower horizontal resolution than is currently 'state-of-the-art', but were integrated for a sufficiently long period (typically $> 10^2$ years at eddying resolution, after a spin-up period of $> 10^3$ years at lower resolution) to allow both the dynamics and the thermodynamics to properly equilibrate.

The numerical model is a standard, primitive-equation Boussinesq z-coordinate model [the Modular Ocean Model (MOM) from GFDL, Pacanowski and Griffies (1999)], although a linear equation of state with no saline effects is used. It was configured in two domains as illustrated in fig. 7. One is an enclosed, rectangular box with wind and buoyancy forcing such as to produce a large subtropical gyre and smaller subpolar gyre. Specifically, the surface temperature is relaxed back to a temperature that is zonally uniform but which diminishes linearly from low to high latitudes. The other domain was constructed so as to represent in a simple way the effects of the Antarctic Circumpolar current. Two sets of integrations were performed; one consists of low resolution, non-eddying integrations with a relatively large value of horizontal diffusivity. These integrations evolved into a completely steady state (except for some small oscillations on the decadal to century timescale) typically after an integration period of a few thousand years. The second set consists of eddy resolving integrations, with a horizontal resolution typically of $1/4°$ or $1/6°$, and a significantly lower value of horizontal diffusivity and viscosity. About 25 vertical levels are used in all integrations. These experiments are typically initialized from an equilibrated lower resolution integration, and then run for an additional hundred years and in some cases much longer. They are vigorously eddying, with eddy kinetic energy several times the mean kinetic energy, and a typical snapshot is illustrated in fig. 8.

A section of the subtropical thermocline in a closed basin in eddying and

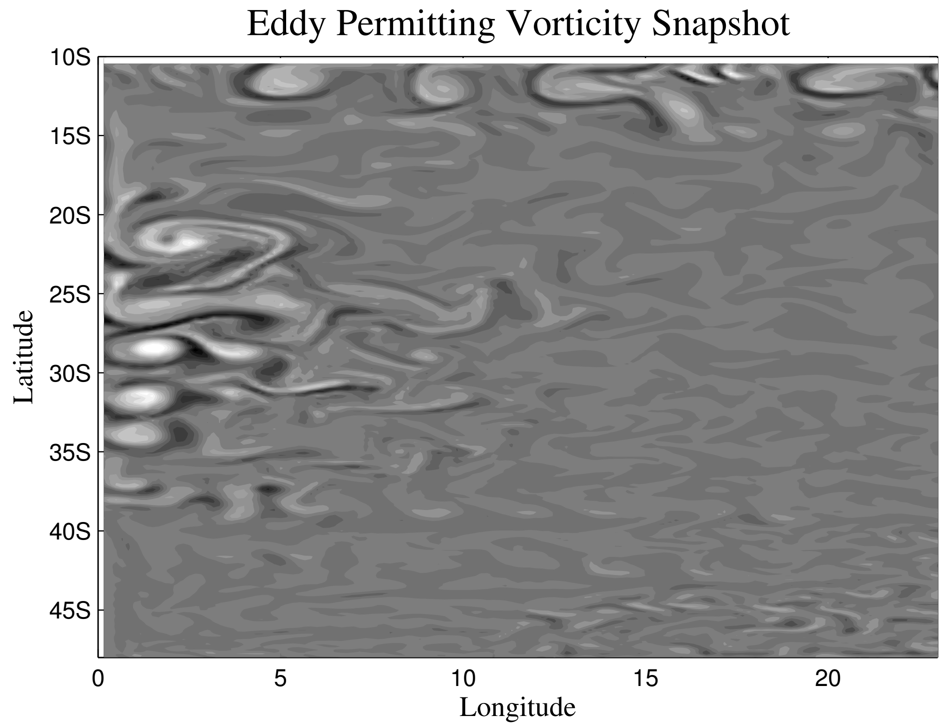

Figure 8. Snapshot of near surface vorticity in an eddying integration.

noneddying runs is illustrated in fig. 9. Qualitatively the structure of the stratification is similar in the two simulations, except in and close to the western boundary layer (not shown). The region of 'mode water' in the noneddying simulations is also partially eroded by the eddies, which are particularly vigorous in this region as the available potential energy of the noneddying state is high. The eddies evidently are quite efficient in extracting this available potential energy, resulting in a more uniform stratification and lowered mean available potential energy. A typical term-by-term analysis of the thermodynamic equation through the center of the subtropical gyre (and a little distance from the western boundary layer) is illustrated in fig. 10. This shows that in the upper thermocline the mean advection terms are still dominant, just as in the classical ventilated picture, with eddy terms of secondary importance and diffusive terms negligible. Obviously, if the eddies were still more vigorous it would quantitatively affect this picture, but the upper thermocline would nevertheless remain an advective regime in the sense that explicit diffusion (i.e. the term $\kappa\partial^2 b/\partial z^2$) is small. However, in the near surface region the effect of the eddies is *diabatic* — that is to say, the eddies induce diapycnal fluxes — as can be seen from the transformed Eulerian mean (TEM) diagnostics of fig. 10. This might be expected because this is a region in which

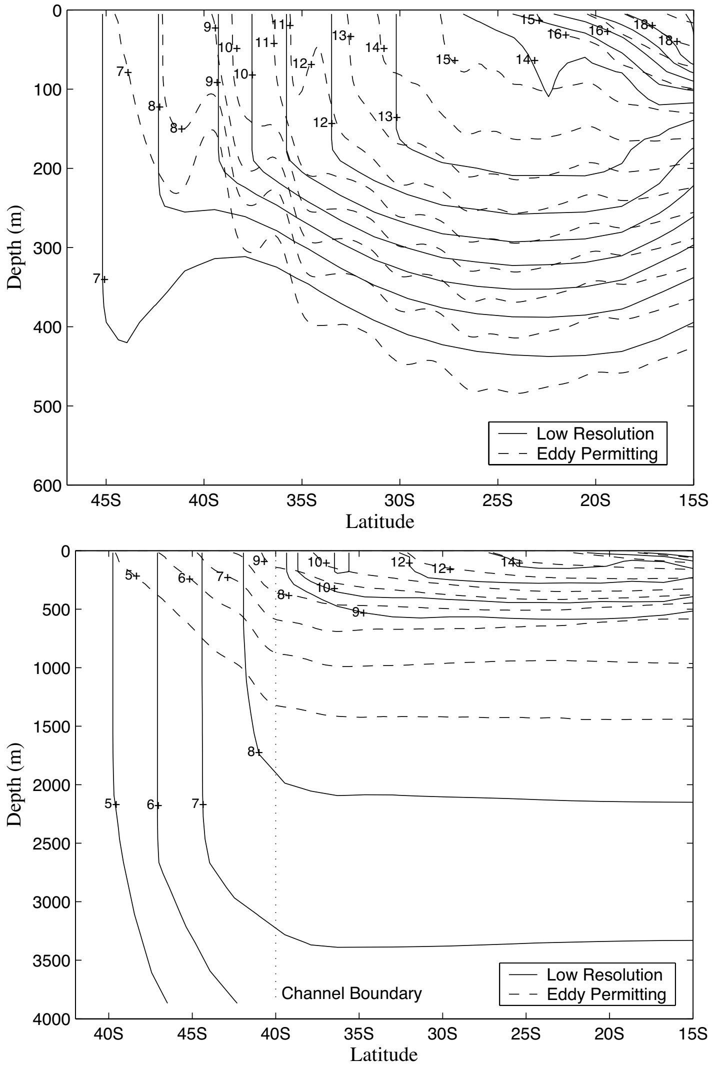

Figure 9. Top: Meridional section of the time averaged temperature in a non-eddying (solid line) and an eddying (dashed lines) simulation in an enclosed box with subtropical and subpolar gyres. Bottom: Similar meridional section, except for an integration with a periodic channel, as in the right-hand panel of fig. 7.

interactions with the atmosphere are important. The lower thermocline has a more complicated balance (in the simulations), involving mean flows, eddy terms and explicit diffusion, although the eddy effects are themselves largely adiabatic. Just as in the non-eddying case, a thermocline base that is distinct from the upper advective regime can still be identified. It is slightly thicker in the eddying case but, interestingly, the thickness still scales with the diffusivity according to $\kappa^{1/2}$,

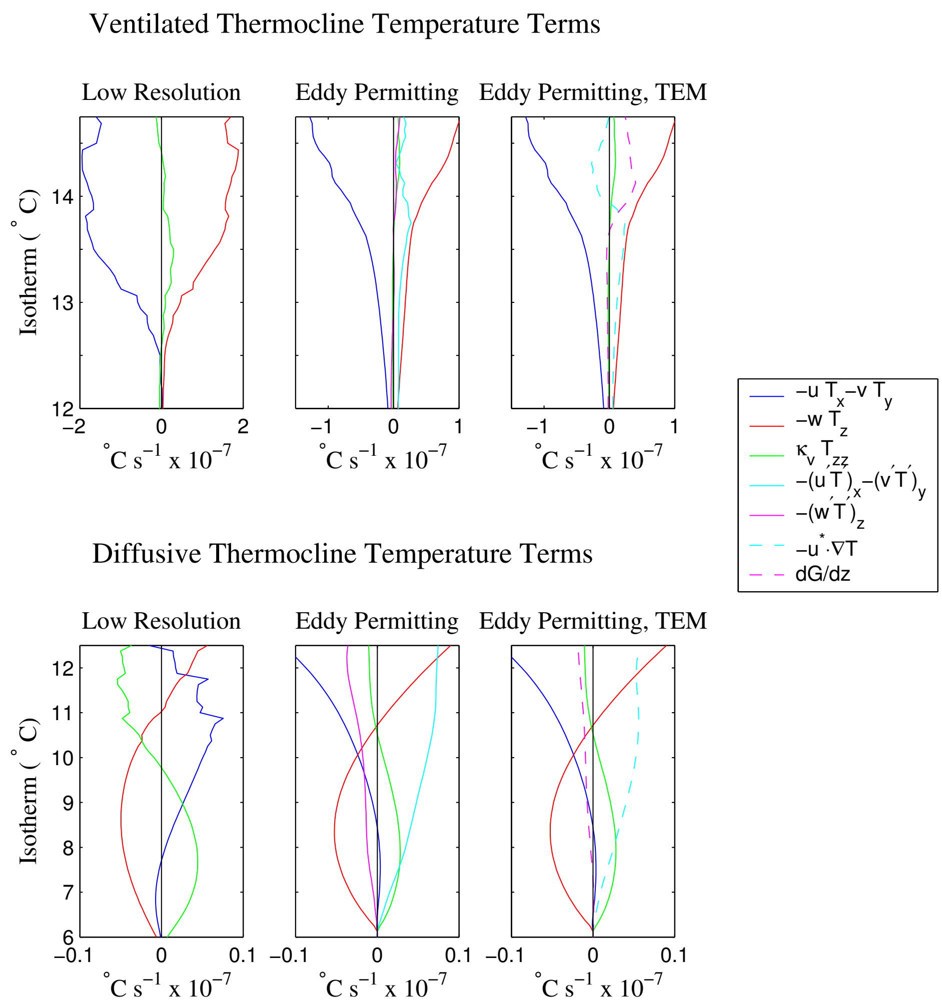

Figure 10. Profile of terms in the thermodynamic equation, plotted as a function of potential temperature, through the middle of the subtropical gyre in eddying and noneddying integrations. The upper portion is for isotherms that outcrop in the Ekman pumping region (ventilated thermocline) and the lower portion is for isotherms that outcrop in the subpolar Ekman suction region (the internal thermocline). The rightmost panels show the transformed Eulerian mean terms: G is the cross-isopcynal eddy buoyancy flux, so that the size of $\partial G/\partial z$ is a measure of the diabaticity of the eddy terms, which evidently is significant near the surface.

much the same way as the noneddying case. It is not known whether this scaling is universal in the eddying case.

Mesoscale eddies tend to have a larger influence in the simulations with a circumpolar channel (fig. 11). The stratification in the channel is quite different with and without eddies and this is at least in part because the non-eddying case has no equivalent of a ventilated thermocline solution, and produces a stratification with near vertical isopycnals that is highly baroclinically unstable. Such instability causes these isopycnals to slump, and the resulting stratification exhibits a balance between the buoyancy input at the surface and its lateral transport by baroclinic eddies [see also Karsten et al. (2002)]. Explicit dissipation (the $\kappa\partial^2 T/\partial z^2$ term) plays no direct role in this, at least locally. The presence of the circumpolar channel greatly affects the adjacent subtropical thermocline, thickening its base and leading to a balance in the thermodynamics between mean advection and eddy fluxes, rather than mean advection and diffusion as in the classical subtropical gyre model. (Gnanadesikan (1999) also noted the effect of the ACC on the deep stratification of the ocean elsewhere.) The thickening might be interpreted as a potential vorticity or thickness transport (and if the beta effect is small, the two are similar) from the circumpolar channel, similar to that illustrated in fig. 6.

5. Concluding Remarks

Several decades of effort have finally yielded a good, although still not complete, understanding of the subtropical thermocline in a world without baroclinic instability; that is, in a world essentially governed by the planetary geostrophic equations. The picture that has emerged is one of an advectively dominated upper thermocline (the ventilated thermocline) and an advective-diffusive base (the internal thermocline), although important aspects of this quasi-laminar picture are still poorly understood — for example the formation and properties of warm pools of low potential vorticity water (‘mode water’). The degree to which this picture should be considered an accurate or quantitative model of the circulation depends to a large degree on the role of mesoscale eddies — do they have only a perturbative effect, or do they completely dominate the circulation, or is it somewhere in between? A definitive answer to this is hard to give, since it will depend at least in part on the results of high resolution numerical simulations as well as observations. The current set of simulations we have described do perhaps allow us a glimpse of the truth, however, and some preliminary conclusions are:

1. In the upper thermocline of the subtropical gyre eddies tend to be most vigorous in or near the western boundary current and in regions of ‘mode water.’ Eddies may be the dominant factor in setting thermocline structure in these regions, although one should remember that it is the structure of these regions as described by noneddying solutions that at least in part leads to the formation of the eddies.

2. Away from the western boundary current and mode water, the signature of the classical ventilated thermocline can be seen, even in strongly eddying regions, at least in the simulations we have performed so far. The lower thermocline exhibits a complex balance involving eddies, mean flow and

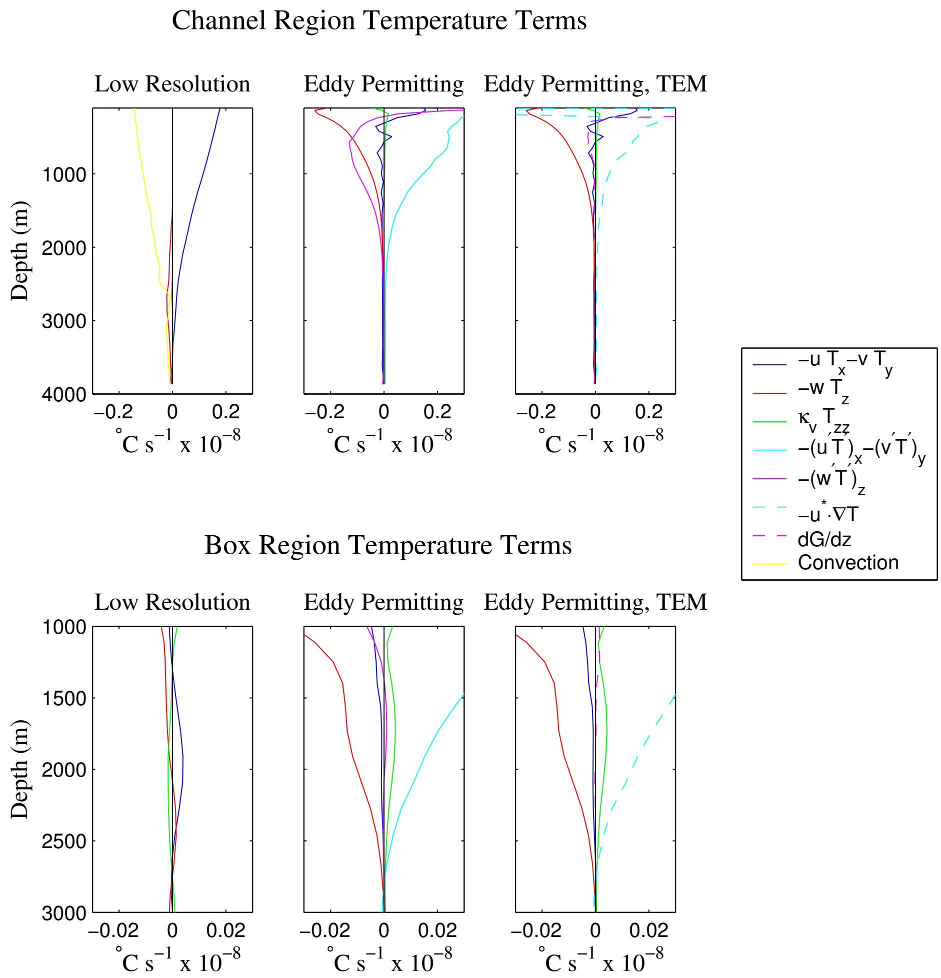

Figure 11. Profile of averaged terms in the thermodynamic equation in the integrations with a circumpolar channel for a low resolution case (left), an eddy permitting case (center) and the eddy permitting case using the TEM form for the eddy terms (right). The upper portion shows the temperature equation terms through the channel region, and the lower portion shows the temperature equation terms in the adjacent subtropical gyre, spanning the depth of those isotherms that outcrop within the channel. Eddy terms are significant nearly everywhere in the channel, and in the lower thermocline of the adjacent subtropical gyre.

diffusion. The importance of eddies is a quantitative issue in these regions, and may differ from basin to basin.

3. Eddies are ubiquitous in simulations with a re-entrant channel, and they are the controlling effect in setting the upper ocean stratification there, strongly suggesting that mesoscale eddies are of overwhelming importance in set-

ting the stratification of the Antarctic Circumpolar Current. The effect of such eddies spreads underneath the adjacent subtropical gyres, thickening the thermocline and reducing the importance of diffusivity in setting thermocline thickness.

4. Near the surface eddies induce diapycnal fluxes and have strong diabatic effects, but in the interior their influence is largely adiabatic.

Even as we answer questions, new ones arise and the general effect of mesoscale eddies on the ocean circulation is very much an open problem. Nevertheless, even without fully solving this grand ‘problem of turbulence’ some more specific problems in ocean circulation may be tractable. Among these are:

1. The thermocline near the western boundary current seems likely to be completely controlled by eddy effects, and we do not have a good understanding of this. Do eddies influence the separation of the western boundary currents? And how does the western boundary current regime transition into the midocean regime, where eddy effects may be secondary?

2. The dynamics of mode water is poorly understood in both noneddying and eddying cases, and it seems likely that an understanding of the latter case will depend on an understanding of the former. In the classical picture the lowest ventilated layer readily forms into a thick thermostad, but this is quite baroclinically unstable and in the eddying simulations we have performed it is apparently partially eroded away. Nevertheless, warm pools of low stratification exist in the real ocean. To understand this we may need to revisit parameterized models, for example Dewar (1986). Perhaps seasonal effects really are important in mode water maintenance?

3. What is the role of explicit diffusion in the interior of an eddying ocean? Eddy effects tend to diminish the role of diffusion in the local balance of terms in the thermodynamic equation, and baroclinic instability can produce its own vertical scale which can be of order 1 km. Yet on more fundamental grounds a finite diffusivity seems necessary to maintain a mean stratification away from a a wind-driven layer, and to produce an overturning circulation.

4. How do eddies affect the abyss? The ‘barotropization’ of energy will lead to large vertical scales and interactions with bottom topography, and indeed eddies may equilibrate via bottom friction. Does deep eddy motion overwhelm the Stommel-Arons circulation? Might the latter still be apparent with very long time averaging?

5. How do mesoscale eddies interact with the mixed layer? Both theoretical reasoning and the numerical results described above suggest that mesoscale

eddies have a diabatic effect in regions where they feel the surface. Perhaps the zeroth order effect of this can be modelled with a simple horizontal (and so cross-isopycnal) eddy diffusion, but a host of complicating factors (predicting mixed layer depth, temperature-salinity compensation, convection, predicting the size of any diffusivity) will make this far from simple.

Let me finish with a couple of subjective comments. First, I hope and expect to see much progress in these areas in the next few years, and some of that may of course determine that the above questions are the wrong ones. Second, I am struck by how important it is to have an understanding of the structure of the ocean in the absence of eddies in order to understand the structure of the ocean with eddies. One might take issue with this comment for the ACC, where the stratification seems largely determined by eddies, but the eddy-free basic state still provides a context for understanding eddy influence.

Acknowledgments
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