Q. J. R. Meteorol. Soc. (1988), **114**, pp. 183–204 551.513.1:551.465

Numerical studies of eddy transport properties in eddy-resolving and parametrized models

By G. K. VALLIS
Scripps Institution of Oceanography, La Jolla, Ca., U.S.A.

(Received 10 March 1987; revised 11 August 1987)

SUMMARY

This paper is a study of the transport properties of large-scale quasi-geostrophic flow, when it is forced by an unstable shear of much greater horizontal scale than the scale of the resultant eddies. One of the purposes is thereby to test certain phenomenological parametrization schemes which predict the transport of potential vorticity or heat, or which predict the time-mean state of the zonally averaged atmosphere. Models used are highly nonlinear spectral quasi-geostrophic models, with sufficient resolution to resolve the energy-containing scales and partially resolve an enstrophy inertial range. A scale separation is enforced between the mean and eddy flow, this being a necessary condition for transfer theories to work. An expression is derived for the rate of change of the mean flow in such cases. As a quantitative predictor of the time-mean state of the zonal flow, baroclinic adjustment is found to work only when nonlinear wave–wave interaction between eddies is small. In such cases, the amplitude of the eddy flow is determined by the forcing on the zonal flow, which itself equilibrates close to the critical value for linear instability. In fully nonlinear models if the zonal flow is strongly forced the mean flow can be highly supercritical. The flux of potential vorticity is found to depend strongly and monotonically on the mean shear, although no obvious simple relationship is found relating it to potential vorticity gradient. In part because potential vorticity is not a passive scalar, using phenomenological transfer coefficients is by no means straightforward. Such transfer relationships are seen to be in some ways unsatisfactory.

1. INTRODUCTION

Two related topics of some importance in atmospheric and oceanic dynamics are the nonlinear equilibrium of baroclinic instability, and the transport properties of fully developed geostrophic turbulence. Baroclinic instability is the principal 'source' of energy for mesoscale oceanic flows and synoptic-scale flows in the atmosphere. By 'source' is meant the mechanism whereby energy is transferred from the mean zonal flow (for the atmosphere) or large-scale gyre flow (for the ocean) into time and spatially varying eddy fields, recognizing that the process is adiabatic. The principles of such transfer are well described by linear theory (beginning with Eady and Charney and described in numerous textbooks). Linear theory by its nature can say nothing about the process of equilibration—this is a nonlinear problem of finite-amplitude baroclinic instability. Equilibration is the process whereby supercritical baroclinic waves nevertheless achieve some steady (possibly only statistically steady) amplitude. At least three mechanisms have been identified as possibilities:

(i) Wave–mean-flow equilibration
(ii) Wave–wave equilibration, via nonlinear energy transfer to linearly stable wavenumbers
(iii) Stochastic equilibration.

(i) *Wave–mean-flow equilibration*: This mechanism essentially involves the mean flow interacting with a single wave, the most unstable wave to small perturbations. This wave produces a correction to the mean flow, possibly of higher meridional wavenumber, such that the wave is in a marginally supercritical state. Some kind of limit cycle would normally ensue. This kind of mechanism can be analytically described using weakly nonlinear theory (e.g. Pedlosky 1970). Some numerical integrations illustrating this are described in section 2.

(ii) *Wave–wave equilibration*: An unstable mode may, via nonlinear interactions, transfer energy to modes of lower wavenumber (or lower pseudo-wavenumber k', $k'^2 = k^2 + \lambda^2$, where k is the wavenumber and λ is an inverse deformation radius for baroclinic modes). By transferring energy, in a cascade, to modes which are baroclinically stable, the unstable modes may equilibrate even at high supercriticalities.

(iii) *Stochastic equilibration*: Baroclinic instability necessitates a correlation between meridional velocity (v) and temperature (T). In a two-layer quasi-geostrophic model the correlation is $\langle(\tau\Psi_x)\rangle$ where τ is the baroclinic streamfunction $\frac{1}{2}(\Psi_1 - \Psi_2)$, and Ψ the barotropic streamfunction $\frac{1}{2}(\Psi_1 + \Psi_2)$, and Ψ_1 and Ψ_2 are the upper- and lower-level streamfunctions and the angle brackets imply a correlation, or normalized time integral. For a linear inviscid problem and uniform zonal flow with $\beta = 0$, one finds $\langle\tau\Psi_x\rangle = 1$, meaning there is perfect correlation between τ and Ψ_x. Both friction and the effects of β reduce this correlation. It has been suggested (Salmon 1980) that turbulence may act as a scrambling mechanism, reducing the correlation and enabling supercritical waves to equilibrate, without the need for net energy transfer.

The above mechanisms all come with many variations. For example, wave–mean-flow equilibration may involve changes in the *vertical* structure of the mean flow. The above mechanisms are described more fully in the papers by Pedlosky (1970), Hart (1979a), Salmon (1980), Loesch (1974), and others.

The mechanism of equilibration, while very interesting *per se*, additionally has obvious ramifications for the transport of heat or potential vorticity in geostrophic turbulence. If wave–mean-flow equilibration is important, then one might expect *baroclinic adjustment* (Stone 1978) to apply. In this scenario, the zonal mean shear builds up due to diabatic heating. As soon as it reaches supercriticality an unstable mode grows, transferring sufficient heat poleward to reduce the instability of the mean flow to a marginally supercritical state. Thus, Stone argues (and presents some empirical evidence to support his case) the time-mean zonally averaged flow of the atmosphere should be close to a marginally supercritical one. Although not couched in the language of nonlinear baroclinic instability, the parametrization clearly depends on mechanism (i) above being dominant. Both mechanisms (ii) and (iii) above allow supercritical equilibration.

It is the purpose of this paper to test experimentally (i.e. numerically) ideas of equilibration, and the transport properties of fully developed geostrophic turbulence. To test these ideas we use the simplest model containing the necessary dynamics—a two-layer quasi-geostrophic model with a spatially (but not necessarily temporally) uniform zonal shear, and examine the dynamics primarily as a function of zonal shear. The advantages of using a model with a uniform zonal shear are the following: a scale separation is enforced between mean and eddy flow, the most favourable condition for transfer/diffusion-like theories to function; the eddy field is homogeneous, which enables certain algebraic results about eddy transfer properties to be obtained straightforwardly; there is no eddy momentum flux, and hence a direct relation between eddy heat and potential vorticity flux. On the other hand, the lack of eddy momentum flux is an unrealistic feature as far as the earth's atmosphere is concerned and obviously no inferences can be drawn about it. However, our main aim is to examine potential vorticity and heat fluxes (since the theories about their transport are the most well founded) and for this purpose the simplest model is preferable. No feature of the model prevents the conclusions we draw about such fluxes being invalid.

The paper is organized along the following lines: Section 2 describes the model formulation. In particular an expression for the rate of change of the mean flow is derived. (Although the expression has been given before, its derivation has always been contrived.

Our derivation relies only on a scale separation between mean flow and eddies. Further, it is valid for multi-layered baroclinic models as well as barotropic models.) Section 3 examines equilibration mechanisms and baroclinic adjustment. Following this, in section 4, we examine the transport properties of fully developed geostrophic turbulence, and particularly the performance of various parametrization schemes (in particular the diffusion of potential vorticity). Although it may appear that the parameter range, from small supercriticality to fully developed turbulence, is large (and hence beyond the sensible scope of a single paper), remember that really only one parameter is to be varied—the mean shear in the zonal wind. Others (e.g. Rossby deformation radius) which would also affect the supercriticality, are fixed. By allowing only the shear to vary, we can explore very different behaviours, governing a fairly full range from laminar to turbulent flow, with a relatively modest set of experiments.

2. Model description

The numerical and analytic tools we use are quasi-geostrophic β plane models in a doubly-periodic domain. Continuous equations may be written:

$$\partial Q/\partial t + J(\Psi, Q) + \beta\, \partial\Psi/\partial x = \text{forcing—dissipation} \tag{2.1}$$

where

$$Q = \nabla^2\Psi + \frac{\partial}{\partial z}\left(\lambda^2(z)\frac{\partial\Psi}{\partial z}\right).$$

Thus Q is the potential vorticity, Ψ is the streamfunction and $\lambda(z)$ is an appropriate inverse deformation radius. It is convenient to consider explicitly a mean zonal flow by writing

$$\Psi = \psi - U(z)y \tag{2.2}$$

and

$$Q = q - y\frac{\partial}{\partial z}\left(\lambda^2(z)\frac{\partial u}{\partial z}\right)$$

where

$$q = \nabla^2\psi + \frac{\partial}{\partial z}\left(\lambda^2\frac{\partial\psi}{\partial z}\right) \tag{2.3}$$

whence, for unforced inviscid flow,

$$\frac{\partial q}{\partial t} + J(\psi, q) + \beta\frac{\partial\psi}{\partial x} = -U\frac{\partial q}{\partial x} + \frac{\partial}{\partial z}\left(\lambda^2\frac{\partial U}{\partial z}\right)\frac{\partial\psi}{\partial z}$$

where for the moment temporal changes in the U field are ignored. If U is held fixed, then the right-hand side provides a source of energy for the eddy field ψ. We shall mainly be concerned with layered models, and for the conventional two-layer model the appropriate equations are:

$$\left.\begin{aligned}\partial q_1/\partial t + J(\psi_1, q_1) + \beta\,\partial\psi_1/\partial x &= -U_1\,\partial q_1/\partial x + \lambda^2(U_2 - U_1)\,\partial\psi_1/\partial x\\ \partial q_2/\partial t + J(\psi_2, q_2) + \beta\,\partial\psi_2/\partial x &= -U_2\,\partial q_2/\partial x + \lambda^2(U_1 - U_2)\,\partial\psi_1/\partial x\end{aligned}\right\} \tag{2.4}$$

where

$$q_1 = \nabla^2\psi_1 + \tfrac{1}{2}\lambda^2(\psi_2 - \psi_1) \qquad q_2 = \nabla^2\psi_2 + \tfrac{1}{2}\lambda^2(\psi_1 - \psi_2).$$

(*a*) *Evolution of the mean field*

With doubly-periodic or channel boundary conditions there has been some discussion, and perhaps some confusion, in the literature about time-varying zonally-averaged mean flows (Salmon 1980; Vallis 1985; Carnevale and Frederiksen 1987). In channel models it appears that a suitable choice of spectral expansion can be found to allow a temporal variation in the mean flow. However, the method does not extend readily to more general geometries, and is computationally rather inefficient (see also Vallis 1985). Below a method for computing temporal variations in a large-scale zonal flow, based on the zonal momentum equation, is presented.

The practical problem, then, is to compute the variation of a zonally-averaged flow (such as $U(z)$ in Eq. (2.2)) which varies spatially on a much larger scale than the eddy flow (although the magnitudes of the velocities may be similar), and on a longer timescale. (The latter is the justification for ignoring the temporal dependency of U in Eq. (2.3).) Thus we choose to expand the streamfunction

$$\psi = (1/\varepsilon)\psi_0(y_0, t) + \psi_1(x, y, y_0, t) + O(\varepsilon) \tag{2.5}$$

where $y_0 = \varepsilon y$ is a large space scale over which the mean flow varies. With such an expression the mean meridional velocity is zero and the corresponding expansion for the geostrophic zonal velocity (u_g) is

$$u_g(x, y, t) = u_0(y_0, t) + u_1(x_1, y_1, y_0, t) + O(\varepsilon). \tag{2.6}$$

We have used

$$u = (\partial/\partial y + \varepsilon\,\partial/\partial y_0)\psi.$$

Similarly,

$$v_g(x, y, t) = v_1(x, y_1, y_0, t) + O(\varepsilon).$$

Thus the mean and eddy zonal velocity fields are similar in magnitude. Note that Eqs. (2.5) and (2.6) are consistent with (but do not necessarily imply) (2.2). One of the aims of the expansion, obviously, is to derive an expression for dU/dt in (2.2), or du_0/dt in (2.6). To do so we begin with the momentum equation in a Boussinesq model. Thus (see Veronis 1981), for each layer i the u momentum equation is

$$\partial u_i/\partial t + \partial(u_i u_i)/\partial x + \partial(v_i u_i)/\partial y - f v_i = \partial p_i/\partial x \tag{2.7}$$

and the continuity equation is

$$\partial h_i/\partial t + (\mathbf{u}_i \cdot \nabla)h_i + h_i \nabla \cdot \mathbf{u}_i = 0 \tag{2.8}$$

where u_i and h_i are the zonal velocity and layer thickness in the ith layer, and p_i is the kinematic pressure. We shall subsequently drop the subscripts i.

The velocity field $\mathbf{u}$ may be written

$$\mathbf{u} = \mathbf{u}_g + \mathbf{u}'$$

where $\mathbf{u}_g$ is the geostrophic component, and $\mathbf{u}'$ the ageostrophic. The layer depth may similarly be written $h = H + h'$ where H is a constant mean thickness. The prime on the variable component of h will subsequently be dropped. Zonally averaging (2.7) gives

$$\partial \overline{u}^x/\partial t + \partial(\overline{uv}^x)/\partial y - \overline{fv'}^x = 0. \tag{2.9}$$

An overbar denotes an average. Primed velocities (e.g. v') are non-geostrophic. Otherwise, all velocities are taken geostrophic. Now, the geostrophic version of (2.8) is

$$\partial h/\partial t + (\mathbf{u} \cdot \nabla)h + H\nabla \cdot \mathbf{u}' = 0$$

since the term $\nabla \cdot \mathbf{u}$ is small, only its contribution when multiplied by the mean layer height H is non-negligible. The second and third terms in the above equation are assumed comparable. The zonal average is

$$\partial \overline{h}^x/\partial t + \partial(\overline{vh}^x)/\partial y + H\,\partial\overline{v}^x/\partial y = 0. \tag{2.10}$$

Inserting (2.6) into (2.9) gives

$$\partial \overline{u_o}^x/\partial t + \partial \overline{u_1}^x/\partial t + (\partial/\partial y + \varepsilon\,\partial/\partial y_o)(\overline{u_1 v_1}^x) - f\overline{v_1'} = 0$$

and integrating over y (not y_o)—i.e. averaging over the eddies—gives, at zeroth order,

$$\partial \overline{u_o}^{x,y}/\partial t - f\overline{v_1'} = 0 \tag{2.11}$$

where we have assumed

$$\overline{u_1}^{x,y} = 0.$$

Now, an expansion for the layer thickness h consistent with Eq. (2.5) is

$$h = (1/\varepsilon)h_o(y_o, t) + h_1(x, y, y_o, t) + O(\varepsilon).$$

Inserting this into (2.10) and integrating over the domain y gives

$$\frac{1}{\varepsilon}\frac{\partial \overline{h_o}^{x,y}}{\partial t} + \frac{\partial \overline{h_1}^{x,y}}{\partial t} + \frac{\varepsilon\partial}{\partial y_o}(\overline{v_1 h_1}^{x,y} + H\overline{v_1'}^{x,y}) = 0. \tag{2.12}$$

We may certainly assume, or require, that $\partial \overline{h_1}^{x,y}/\partial t = 0$, i.e. that there is no mean change in the eddy layer thickness field. Now integrating (2.12) over y_o gives

$$\overline{v_1 h_1}^{x,y} + H\overline{v_1'}^{x,y} = 0 \tag{2.13}$$

where a zero value for the constant assumes no boundary or far field forcing. Substituting (2.13) in (2.11) finally gives the equation for the mean flow

$$\partial u_o/\partial t - (f/H)\overline{vh}^{x,h} = 0. \tag{2.14}$$

This is the final, and unsurprising, result. It means that the mean flow in each layer is affected by the effective form drag, either through topography or variations in dynamic height. The result could almost have been written down from Eqs. (2.10) and (2.11). The main assumption in the derivation is a scale separation between the mean flow and the eddy field and hence that eddy momentum flux divergence is zero. Given this, the derivation merely assures that (2.14) is consistent. Note that (2.14) allows both the barotropic and baroclinic components of the mean field to vary, since the result holds for every model layer separately. No meridional walls are necessarily required. The result may also be derived by considering the integrated pressure, or form, drag of an obstacle h in a zonal stream, although then the momentum flux divergence would have to be ignored more arbitrarily.

In a quasi-geostrophic model the expansion (c.f. (2.2))

$$\Psi_i = -U_i y + \sum \psi_{i,\mathbf{k}}\, e^{i\mathbf{k}\cdot\mathbf{x}} \tag{2.15}$$

satisfies the scale separation assumption. Effectively, we have relegated all the very large scale (wavenumber <1) changes in the streamfunction field to changes in Uy. For a two-layer model one finds

$$\left.\begin{aligned} dU_1/dt &= -\lambda^2[(\psi_1 - \psi_2)\psi_{1x}] \\ dU_2/dt &= -\lambda^2[(\psi_2 - \psi_1)\psi_{2x}] + [h\psi_{2x}] \end{aligned}\right\} \quad (2.16)$$

where a square bracket denotes an integration over the domain, h is the static surface topography and $\psi_x = \partial\psi/\partial x$. Note that with $h = 0$,

$$d(U_1 + U_2)/dt = 0$$

consistent with Galilean invariance. Equations (2.16) imply energetic consistency, in the sense that a measure of the total energy is conserved. Explicitly, for a two-level model in the absence of topography

$$\frac{d}{dt}[U^2 + (\nabla\psi)^2 + (\nabla\tau)^2 + \lambda^2\tau^2] = 0$$

and

$$U = (U_1 - U_2)/2, \quad \tau = (\psi_1 - \psi_2)/2, \quad \psi = (\psi_1 + \psi_2)/2.$$

This generalizes to multi-layers with topography.

For the rest of this paper we use exclusively a two-layer model with no bottom topography. Thus we integrate (2.4) and (sometimes) (2.16) on a doubly-periodic domain. The code is a de-aliased spectral code. Friction is included by adding to the right-hand sides of (2.4) the terms $-r\nabla^2\psi - \nu\nabla^6\psi$, for $i = 1$ and 2. In dimensional units r has an approximate value 1/10 day^{-1}. The parameter ν is made proportional to the square root of the total enstrophy (so inversely proportional to the eddy turnover time at the smallest scales) to remove enstrophy appropriately. Note that friction is included symmetrically in upper and lower levels. This is unrealistic as far as the earth's atmosphere goes, but simplifies the model and does not corrupt the essential, inviscid, dynamics.

3. Equilibration mechanisms

In this section we examine equilibration of supercritical baroclinic waves. We perform sets of experiments (see Table 1) at relatively modest resolution ($k_{max} = 16$, giving an equivalent 32 grid points in each horizontal direction). The important mechanisms of equilibration occur close to the Rossby deformation radius (wavenumber 8) so maximum wavenumber 16 is certainly adequate. Test cases were performed at double and quadruple the resolution, with minor quantitative differences.

TABLE 1. List of experiments and certain parameters

Experiment	Zonal flow	k_β	U	Asymmetric modes
V1	Variable	5·9	2·8	All
V2	Variable	8·1	1·48	(6,0)
V2b	Variable	8·1	1·48	(6,0)
V3	Variable	7·0	2·0	(6,0),(3,3),(9,3)
V4	Variable	6·2	2·5	(6,all),(0,all)
F1	Fixed	4·5	4·8	All
F2	Fixed	5·0	3·9	All
⋮	⋮	⋮	⋮	⋮
F9	Fixed	8·0	1·52	All

The U and k_β values are the time-averaged values for the experiments with zonal flow variable.

(a) *Baroclinic adjustment*

To test baroclinic adjustment we performed experiments allowing U to vary according to

$$dU/dt = \lambda^2 \overline{\tau\psi_x}^x + r(U^* - U) \tag{3.1}$$

where U^* is the forcing shear, r is the damping timescale, and $U = U_1 = -U_2$. Baroclinic adjustment would imply that the time average of U is close to the critical level for linear instability.

The first set of experiments uses a fully nonlinear model, with both wave–wave and wave–mean-flow interaction. Figure 1 shows a time series of U, for $U^* = 6U_c$. Clearly the value is above the critical level, i.e. there is supercritical equilibration. If baroclinic adjustment were to occur, the flux $\langle V\tau \rangle$ would have to be very small below supercriticality, increasing rapidly above it. This is looked at further in section 4. To test the effects of the forcing term, U^* was reduced to be about $2U_c$. The time-averaged zonal flow (not shown) then turns out to be 1·5 U_c—still supercritical but less so.

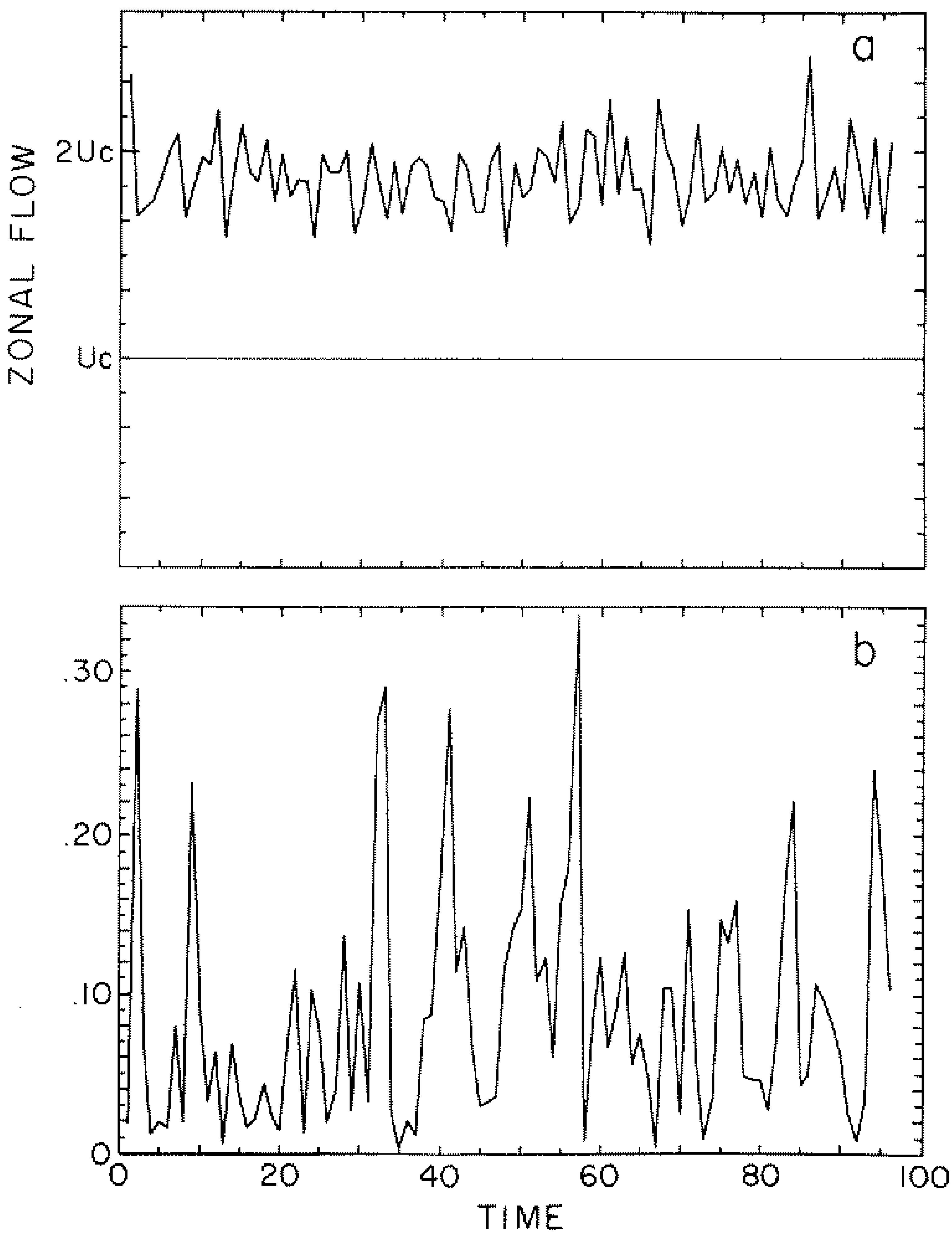

Figure 1. (a) Time series of the zonal velocity in experiment V1. The eddy field is fully nonlinear and interactive. U_c is the critical value for linear baroclinic instability. The final equilibrium is supercritical. (b) Time series of amplitude of (6, 0) mode.

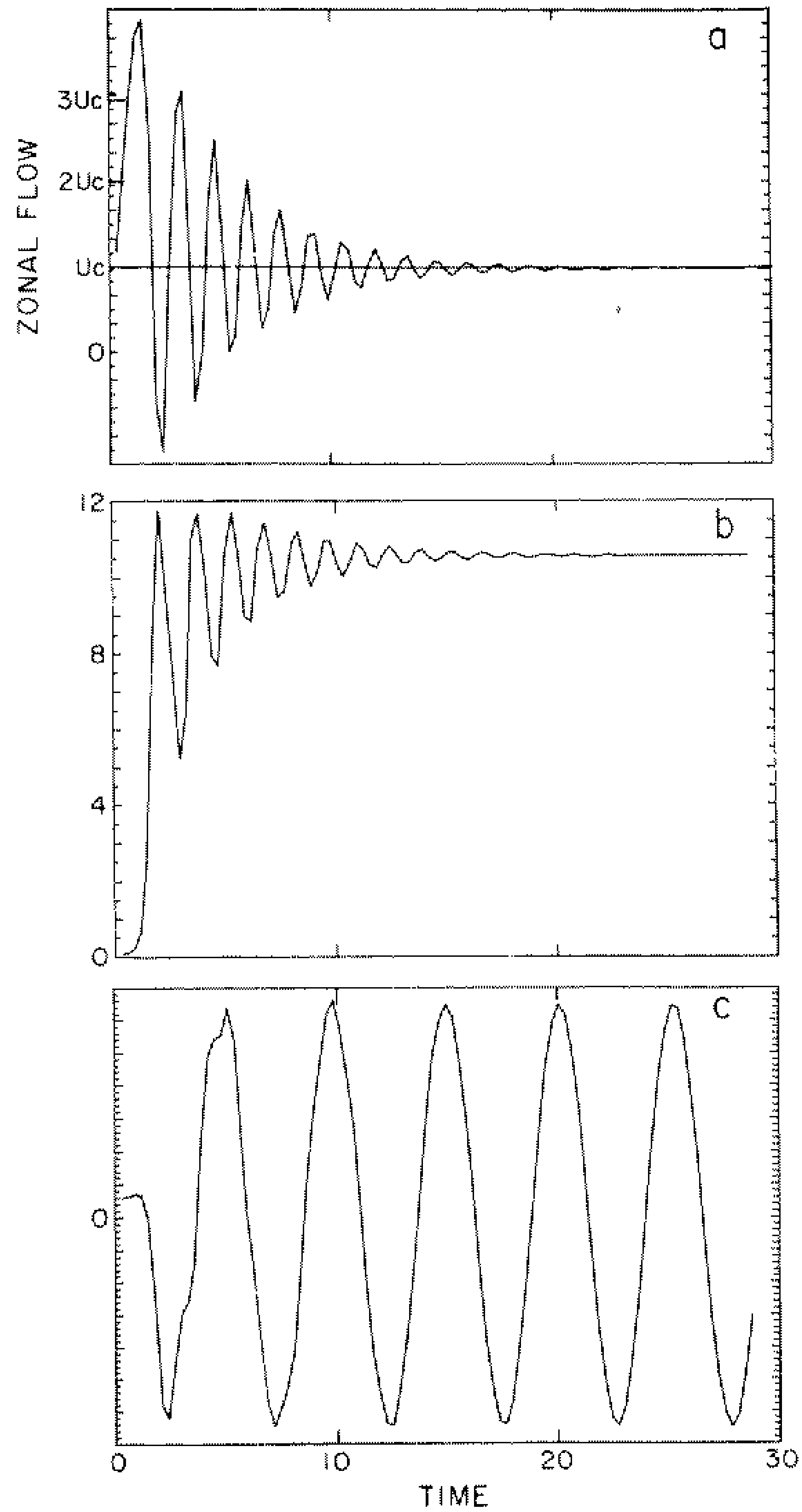

Figure 2. (a) Evolution of U in experiment V2. Only one eddy mode is allowed, (6, 0). (b) Evolution of amplitude of the eddy mode. (c) Evolution of sine component of the eddy mode. The final state is a limit cycle of constant amplitude.

Integrations were carried out to examine the effects of nonlinearity. If only one asymmetric mode is allowed to be non-zero, the only possible equilibration mechanism is wave–mean-flow interaction. Indeed, numerical integration does show the mean flow equilibrating at a level close to criticality (Figs. 2 and 3). If the initial value of the zonal flow is small it will grow until linearly unstable. The magnitude of the 'eddy'—wavenumber $(k_x, k_y) = (6, 0)$—may then grow. This particular wavenumber is chosen because linear analysis shows the mean zonal flow to be the most unstable to small perturbations at this wavenumber. Energy is transferred to the asymmetric flow, which subsequently grows in an oscillatory way. After many oscillations the zonal flow finally equilibrates precisely at the critical level for instability—it is *not* determined by the zonal forcing. The zonal flow is in fact undergoing very small oscillations about the critical value; the amplitude of the oscillations will be determined by the forcing on it. The total magnitude of the eddy flow is virtually constant, although its sine and cosine components

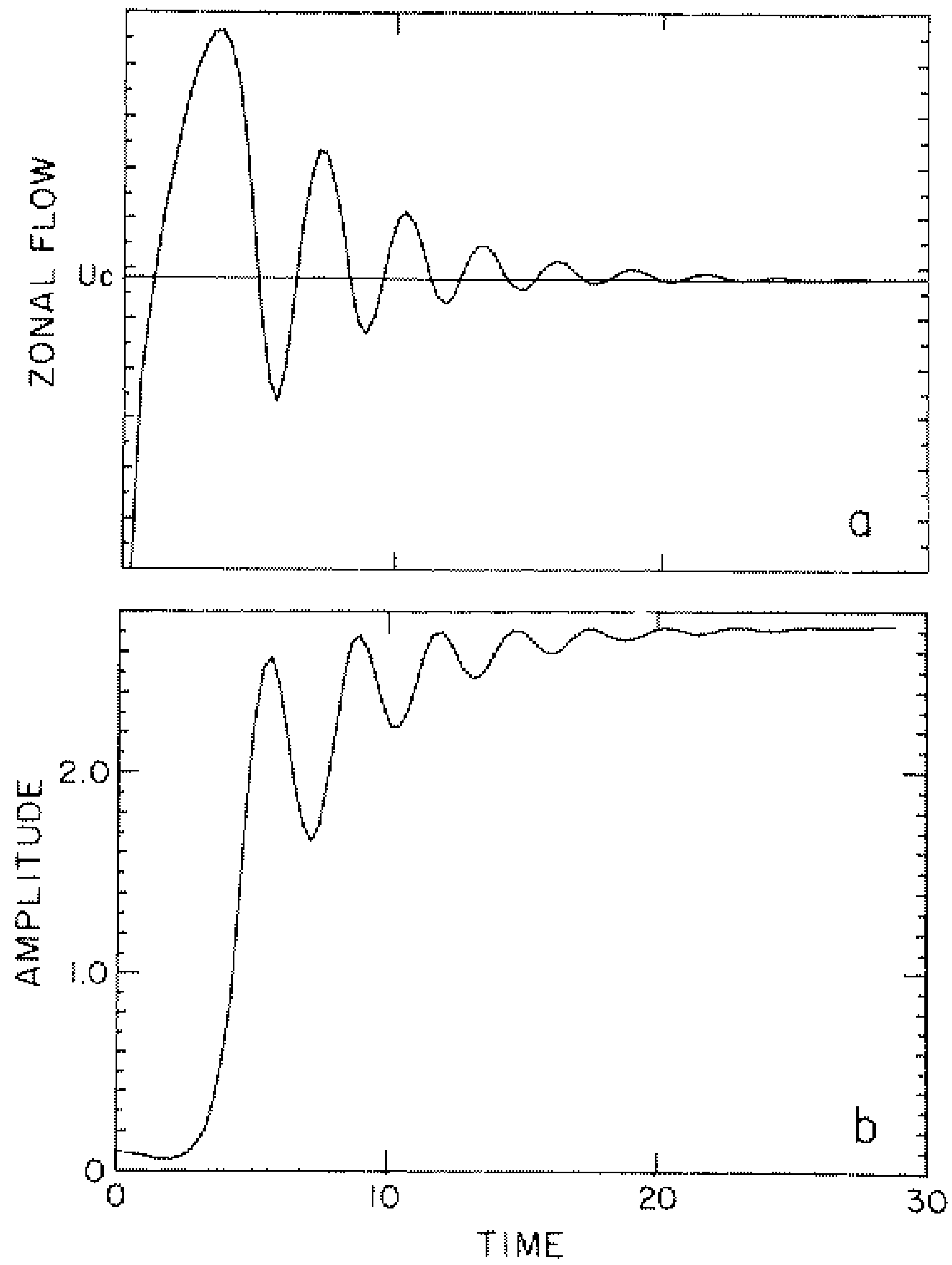

Figure 3. (a) Evolution of U in V2b. The forcing U^* is smaller than in V2, but the final equilibrium value is the same, namely the critical value. (b) As Fig. 2(b) but for V2b. Note the magnitude is much smaller than in 2(b). The units, although arbitrary, are the same as in Fig. 2 (although the scale of the ordinate differs).

oscillate. The magnitude of the eddy flow *is* determined by the forcing on the mean flow, which it feels through the small deviations of the zonal flow from criticality. If the forcing on the zonal flow is reduced, the equilibrium value of the zonal flow is more or less unaltered, but the eddy amplitude is reduced (Fig. 3). (This is similar to the situation in simple models of flow over topography wherein the value of the zonal flow can be locked close to resonance, almost irrespective of the forcing on it, whereas the magnitude of the eddy, or zonally asymmetric, flow is determined largely by the zonal forcing.) The mechanism occurring here is precisely baroclinic adjustment. An experiment was performed in which all wavenumbers were allowed to exist, but the nonlinearity limited to wave–mean-flow interaction and wave–wave interaction is suppressed. With an initial zonal state supercritical to a range of wavenumbers, then all of those wavenumbers initially grow, extracting energy from the mean flow. The most unstable wave grows fastest, and as the mean shear falls some wave vectors become stable again. Ultimately, the shear falls to such an extent that only one wave vector is unstable, and all others decay to zero. This takes a considerable time (dimensionally several months) but ultimately the behaviour is as if only one asymmetric mode were allowed. Thus it is the effects of wave–wave interaction which appear to obviate the need for baroclinic adjustment, rather than the presence *per se* of many asymmetric modes. We examine this further below.

(*b*) *Nonlinear equilibration*

The artificiality of the wave–mean-flow problem lies in the inhibition of a secondary instability. As the forcing on the zonal flow increases from zero the first instability is the usual baroclinic instability of the zonal shear. Only one asymmetric mode initially becomes unstable, we shall refer to this as the primary wave. As the forcing further increases, the amplitude of the primary wave (the most unstable mode of the asymmetric flow) increases also, but the zonal shear is held fixed at the critical shear, since no other equilibration mechanism (other than wave–mean-flow interaction, or indeed 'baroclinic adjustment') exists. At some critical value the primary wave will become unstable to further wave–wave interactions. Since the amplitude of the zonal flow is more or less fixed, this will happen *before* the zonal flow becomes unstable to other asymmetric modes. In general the primary wave will become unstable to a triad interaction involving two other wave modes, one of larger scale and one of smaller scale. The most unstable wave will then be able to transfer energy and enstrophy to these secondary modes. Beyond this secondary instability, there is no reason to expect baroclinic instability to hold. Indeed we might (*extremely* crudely) parametrize the effects of the secondary waves on the primary wave as a damping effect, or a friction, since they are withdrawing energy from it. Any friction added to the primary wave will (unless it is very small, in which case it can destabilize the system) increase the shear of the zonal flow required for instability, allowing the zonal flow to equilibrate at a level supercritical to the old linear stability threshold. Thus we see that nonlinearity can immediately allow supercritical equilibration, if the supercritical level is defined using a linear criterion.

There are two simple systems which allow for nonlinear effects. One is a single triad of interacting waves, plus the zonal flow. The second allows only those modes with a given x wavenumber (say wavenumber 6 in our system, since this is normally the most linearly unstable) but allows all the meridional modes at that zonal scale. Both of these are to some extent ad hoc choices, although both have been used in weakly nonlinear theories of wave–mean-flow interaction (see Loesch (1974) – although Loesch's triads

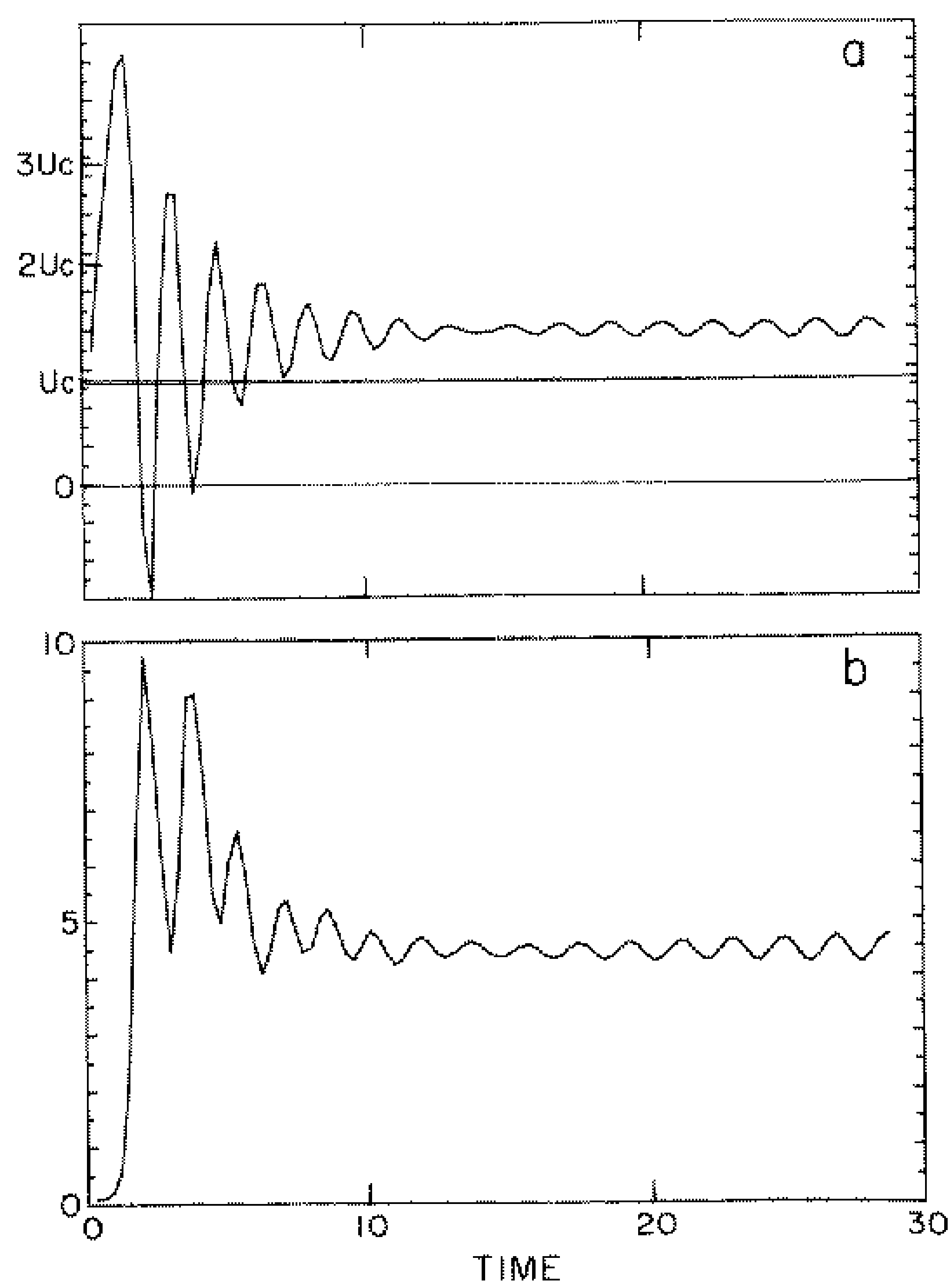

Figure 4. (a) Evolution of U in triad experiment V3. This experiment keeps wavelengths (6, 0), (3, 3) and (9, 3). The forcing is the same as that for V2 (Fig. 2). The final state is supercritical. (b) Evolution of the eddy mode (6, 0)—the same as the eddy mode in V2 (Fig. 2(b)). Note that the average amplitude is lower than that in V2.

were resonantly interacting, a condition we do not impose here) and both illustrate the primary nonlinear interactions occurring. For the first class we choose wavenumbers $(k_x, k_y) = (6,0)$, $(3,3)$ and $(9,3)$. This choice is governed by the desire to have modes with wavenumbers both above and below the most linearly unstable wave. Figure 4 illustrates the time evolution of the zonal flow. Even with this limited nonlinearity, the flow does not settle down to a completely steady state, but oscillates. The time-averaged state is seen to be supercritical. For the second problem Fig. 5 shows the corresponding time series. The zonal flow is supercritical and oscillates chaotically. Even though the wavenumber $(6,0)$ is nominally the smallest, its pseudo-wavenumber k' for its baroclinic mode is higher than the barotropic wavenumbers of higher meridional modes. Thus energy transfer can still take place to larger scales. For both integrations the magnitude of the primary asymmetric mode, $(6,0)$, is smaller than that for the wave–mean-flow problem.

4. Transport in fully nonlinear geostrophic flow

This section examines the transport properties of geostrophic turbulence at high supercriticalities, in a fixed spatially uniform shear. Primarily, our interest is in the meridional transport of potential vorticity and, by default, heat. Secondarily, we shall also be interested in the properties of highly truncated but still nonlinear models.

(*a*) *Austauch coefficients in a two-layer model*

Austauch coefficients are often used to model transport, one main assumption necessary being a scale separation between mean and eddy fields. Green (1970) made the pertinent observation that it is only valid (if it is ever valid) to treat closely conserved

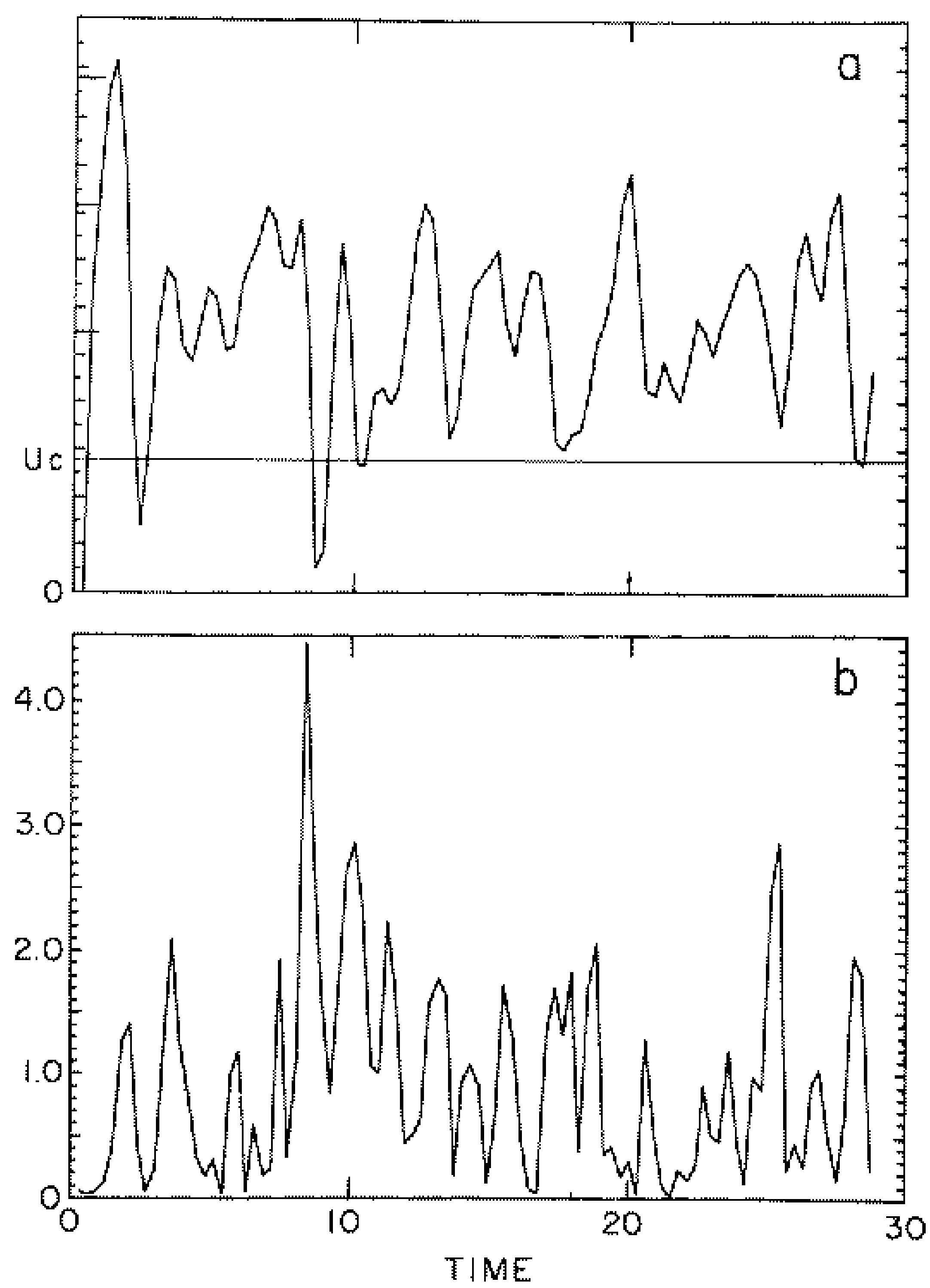

Figure 5. As Fig. 4 except for V4. This experiment keeps all eddies with x wavenumbers 6 or 0.

quantities—such as potential vorticity, q—in this manner. See also Rhines and Young (1982). Let us assume we may write

$$\overline{u_i' q'} = -K_{ij}\, \partial\bar{q}/\partial x_j \tag{4.1}$$

where K_{ij} is a second rank tensor. Dynamically, we often only require eddy flow divergence (with ramifications discussed by Marshall and Shutts (1981)). In that case, only the rotational part of $\overline{\mathbf{u}'q'}$ is dynamically active, and a uniform gradient of q (e.g. β) is also dynamically inactive, provided K_{ij} is uniform. For our problem, Eq. (2.14) shows that it is just the meridional flux of potential vorticity, the form drag, that affects the zonal flow. Since our problem is homogeneous, being doubly-periodic geostrophic turbulence with a uniform zonal shear, K_{ij} *must*, if it exists, be uniform. Furthermore, there can only be a meridional, and no zonal, transport. Hence we suppose

$$\overline{v'q'} = -K\, \partial\bar{q}/\partial y$$

where K is a spatially uniform scalar which will be called a transfer coefficient and the overbar denotes either a time mean or a zonal mean (they are equivalent, with any sort of ergodic hypothesis, because of the homogeneity of the problem). Because of this homogeneity, there can be no meridional momentum flux. To see this, note that

$$\overline{v'\zeta'}^{x} = \partial(\overline{u'v'}^{x})/\partial y$$

where $\zeta = \nabla^2\psi$ is the local vorticity. Integrating this with respect to y with periodic boundary conditions immediately gives $\overline{v'\zeta'}^{x,y} = 0$. (This result is true in a channel also.) But, by homogeneity, $\overline{v'\zeta'}^{t}$ is the same everywhere. Hence $\overline{v'\zeta'}^{t} = 0$ and there is no momentum flux convergence on average, anywhere. This constraint has important ramifications. Consider, for example, a one-layer model of uniform flow over topography. Then,

$$\partial U/\partial t \propto \overline{v'q'} = \overline{v'h'}$$

where h is the topography, which we suppose to have no mean component. Using the parametrization

$$\overline{v'q'} = -K\partial\bar{q}/\partial y = -K\beta \tag{4.2}$$

would clearly not generally be valid. If the topography is random, with no mean gradient, then the potential vorticity flux is parametrized as $K\beta$, which is obviously zero for $\beta = 0$. This is acceptable if the flow is inviscid (by the Charney–Drazin theorem. Also, the form drag of the steady asymmetric response to uniform flow over topography vanishes as viscosity vanishes). However, if the viscosity is non-zero, there will be a form drag, as in the Hart problem (Hart 1979b), and contrary to (4.2). Thus, Austauch coefficients are inappropriate in the simplest of all cases—one-layer flow. In a two-layer model a meridional heat flux is maintained by the mean shear. First, let us establish a few identities about the mean flow:

$$\overline{q_1} = \beta y + \tfrac{1}{2}\lambda^2 y(U_1 - U_2) = \beta y + \lambda^2 U y$$
$$\overline{q_2} = \beta y - \tfrac{1}{2}\lambda^2 y(U_1 - U_2) = \beta y - \lambda^2 U y$$

then

$$\left.\begin{aligned}\partial\overline{q_1}/\partial y &= U(k_\beta^2 - \lambda^2)\\ \partial\overline{q_2}/\partial y &= U(k_\beta^2 + \lambda^2)\end{aligned}\right\} \tag{4.3}$$

where $k_\beta = \sqrt{(\beta/U)}$. Now consider the eddy fluxes. We have

$$\overline{v_i' q_i'} = \overline{v_i' \zeta_i'} + \overline{v_i' \theta_i'} \qquad i = 1, 2$$

where

$$\zeta_i = \nabla^2\psi_i \quad \text{and} \quad \theta_i = \tfrac{1}{2}\lambda^2(\psi_j - \psi_i) \qquad (j = 3 - i).$$

But $\overline{v_i'\zeta_i'} = 0$, for all i, and $\overline{v_1'\theta_1'} = \overline{\psi_{1x}\psi_2} = \overline{-v_2'\theta_2'}$. Hence

$$\overline{v_1'q_1'} + \overline{v_2'q_2'} = 0$$

and

$$\overline{v_1'q_1'} = \overline{v_2'\theta_1'} = -\overline{v_2'q_2'} = -\overline{v_2'\theta_2'}. \tag{4.4}$$

Note that θ is proportional to the quantity h in (2.14). Thus the meridional fluxes of heat and potential vorticity are equivalent. These identities impose constraints on any Austauch coefficients for heat or potential vorticity transfer. Let

$$\overline{v_i'q_i'} = -K_i\,\partial\overline{q_i}/\partial y$$

Then

$$\overline{v_1'q_1'} = -K_1U(k_\beta^2 + \lambda^2)$$

$$\overline{v_2'q_2'} = -K_2U(k_\beta^2 - \lambda^2)$$

whence, by (4.4)

$$K_1/K_2 = (\lambda^2 - k_\beta^2)/(\lambda^2 + k_\beta^2). \tag{4.5}$$

(These are similar to, but even simpler than, relationships in Marshall (1981).) Note that if both K_1 and K_2 are to be positive, $\lambda^2 > k_\beta^2$, which is also the condition for linear instability. Defining $K_o = (K_1 + K_2)/2$ one finds

$$\overline{v_i'q_i'} = \overline{v_i'\theta_i'} = -K_oU(\lambda^4 - k_\beta^4)/\lambda^2. \tag{4.6}$$

Interestingly enough this implies, even with constant K_o, a rapid increase of heat flux with supercriticality.

It seems, therefore, that with a suitable choice of Austauch coefficients in the various layers the required momentum constraints can be satisfied. This procedure was followed by White and Green (1982, 1984) and Marshall (1981). It is obviously necessary to obey the constraints (although some other workers have not done so) since they are identities and momentum conservation is otherwise violated. Indeed, the constraints may be thought of as advantageous, in giving otherwise unknown information about the transfer coefficients. However, it is equally possible to argue that it is a rather arbitrary procedure. In the model above, the potential gradient differs in the two layers, yet we know the magnitude of the potential vorticity flux to be the same. Although this may give useful information about any putative transfer coefficients, it indicates a difficulty in basing transfer on the gradient of potential vorticity. In particular, note that where the vertically integrated potential vorticity flux vanishes, the vertically integrated potential vorticity gradient does not, being given by the β term in the equations of motion. There is no *a priori* reason why the transport theory should not be used for the vertically averaged problem, but clearly it would be inappropriate to do so, for imposing the necessary constraints would give a zero value for the Austauch coefficient. The problem has arisen because potential vorticity is not a passive scalar. Because there can be no momentum flux, potential vorticity is related to the heat flux which takes place at the layer interface. This forces the transfer coefficients to be different in the two layers. If the β term were ignored in the parametrization, then the problem would disappear, although then the potential vorticity would no longer necessarily be transported downgradient.

Consider the case of a multi-layered model, with a constant uniform shear. Again the vertically integrated potential vorticity flux must be zero. This imposes a single constraint on the transfer coefficients, which one could use to give the overall vertical structure. One may also suppose that the average value is given to be the same as that for the two-level model (although we have seen that this value is actually inappropriate for a one-level model, as the value there must be zero). Nevertheless, the detailed vertical structure must now be given by other arguments, whereas for the two-layer model no other information is necessary. This is not necessarily inconsistent, but it seems somewhat unsatisfactory.

The above examples have illustrated that diffusion of potential vorticity using Austauch coefficients is not an entirely self-contained parametrization in the sense that the coefficients themselves must be set according to the vertical resolution of the model used, even though potential vorticity is only being horizontally diffused. The continuous form to which the coefficients in the layered models presumably converge in the limit of infinite vertical resolution is not given, and the limiting case for the most coarse resolution (namely just one layer in the vertical) is quite different from the average value when two layers are used. Of course, this is *not* to say that potential vorticity is not transferred downgradient. Indeed, this is probably the sense of most such eddy transfer in both atmosphere and ocean. Nor do the above arguments necessarily imply that using transfer coefficients in low-order, highly parametrized models is inappropriate. In many ways, they have proved to be quite successful, as in White and Green (1982), and indeed the vertical structure implied by Eq. (4.5) is similar to that implied by the linear arguments of Green (1970). See also Held (1975). The important, and still unanswered, question, is when is it appropriate for such transfer to be so parametrized, or, presumably, when does potential vorticity behave as a passive scalar? Obviously considerations of potential vorticity dynamics (Green 1970; Marshall 1981; Rhines and Young 1982; and others) have been very important in elucidating the structure of the time-mean flow of both atmosphere and ocean. The fact that Green and others are able to obtain variations of the zonal flow qualitatively and even quantitatively similar to that observed is strong evidence indeed that the parametrizations contain much that is correct. It may be that in inhomogeneous flows the sense of the potential vorticity flux is all that is required and this transcends the difficulties mentioned above.

Pressing on regardless, we shall explore further the parametrization schemes. Because of the simple geometry of our problem, Eq. (4.5) may be regarded as the definition of an Austauch coefficient, K_0. The sole problem is to determine its magnitude. Since we have seen that supercritical equilibration does occur, we cannot use a measure based on the total energy available in the mean flow, since we have seen that not all this is used or converted to eddy energy. Indeed, we have seen that in low-order systems the eddy magnitude is determined more by the magnitude of the forcing than by the amplitude of the zonal flow. (This is not strictly accurate, since the eddy motions do not directly feel the forcing on the zonal flow; rather, the eddy flow in that case is in reality extremely sensitive to the magnitude of the zonal shear.) We shall now present some numerical simulations.

(*b*) *Transport properties*

For each simulation the time-averaged meridional potential vorticity (heat) transport was calculated. Figure 6 shows the transport for various measures of the supercriticality. We see that transport increases very rapidly indeed. Even using the formulation of Eq. (5.5), using a constant value for the transport coefficient would clearly be an error. Figure 7 shows how the transport coefficient does vary across the experiments. Clearly

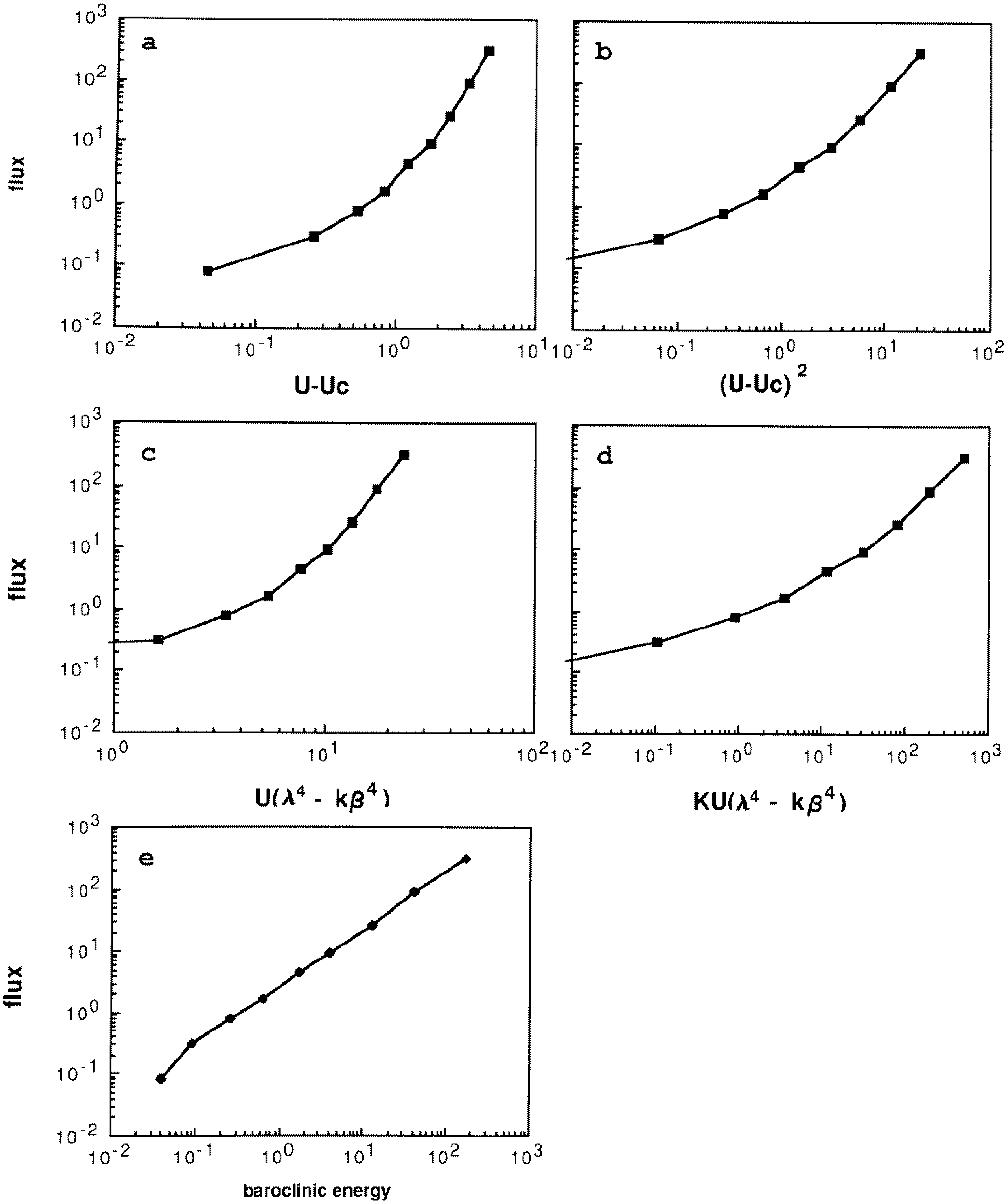

Figure 6. Amplitude of the total potential vorticity flux for various measures of the supercriticality in experiments F1 through F6. The abscissae are: (a) supercriticality $U - U_c$; (b) $(U - U_c)^2$; (c) $K_o U(\lambda^4 - k_\beta^4)$, with K_o constant. This is the right-hand side of Eq. (4.6). (d) $K_o U(\lambda^4 - k_\beta^4)$, with $K_o = (U - U_c)^2$. The latter is meant to give to the transfer coefficient a measure of the APE available in the mean flow. (e) The average total eddy baroclinic energy. The critical shear for linear instability has a value of 1·48 in these units, corresponding to a value of k_β of 8·1. (The respective inviscid values are 1·5 and 8·0. Friction is actually acting to reduce the critical shear here, because it is small.)

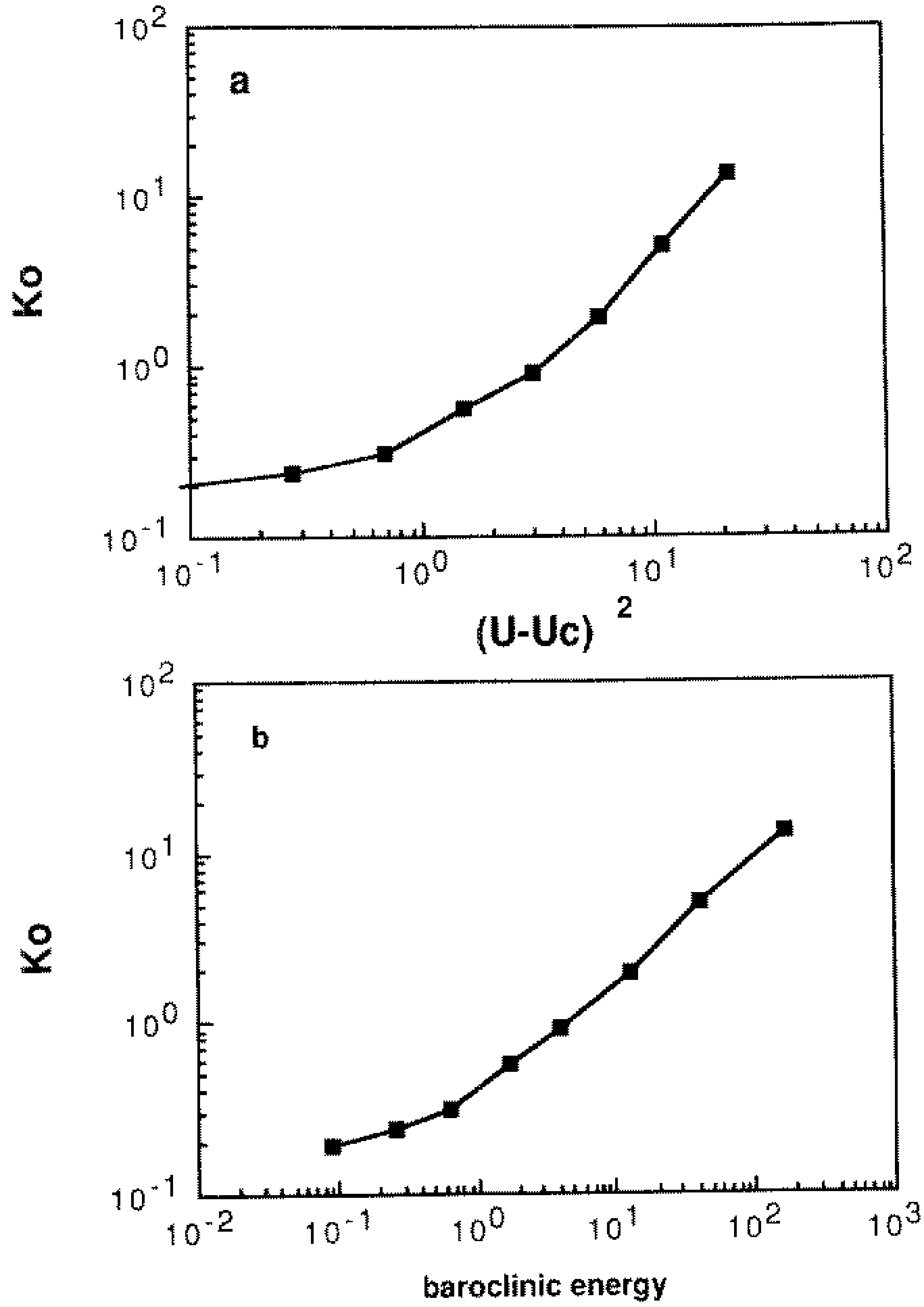

Figure 7. Empirical value of K_o as a function of: (a) $(U - U_c)^2$; (b) average total baroclinic eddy energy. K_o is defined as the flux divided by $U(\lambda^4 - k_\beta^4)$.

some kind of energy weighting is appropriate, but the precise form is by no means clear (as in Harrison 1978). An interesting diagnostic is the correlation of temperature with northward velocity. This is the means whereby heat and potential vorticity are transported polewards. The heat flux as a function of wavenumber is shown in Fig. 8. Not surprisingly, the flux occurs mainly in the energy-containing eddies although they are not necessarily baroclinically unstable to the mean flow. For the linear problem, a correlation occurs only at the wavenumbers which are linearly stable, and for these wavenumbers the correlation is fairly high. For the nonlinear integration, the correlation is generated for a much larger band of wavenumbers, by nonlinear interactions, but it is reduced at the linearly unstable modes from the linear values. This is an explicit example of turbulent scrambling. A value of the correlation less than unity reduces the efficiency at which APE is withdrawn from the mean shear.

(c) *Subgrid-scale parametrizations for medium resolution models*

The transport theories discussed above (namely baroclinic adjustment and potential vorticity transport) are meant to apply to the modification of a mean flow by eddies of a much smaller scale. The difficulties associated with the transport schemes may be summarized as:

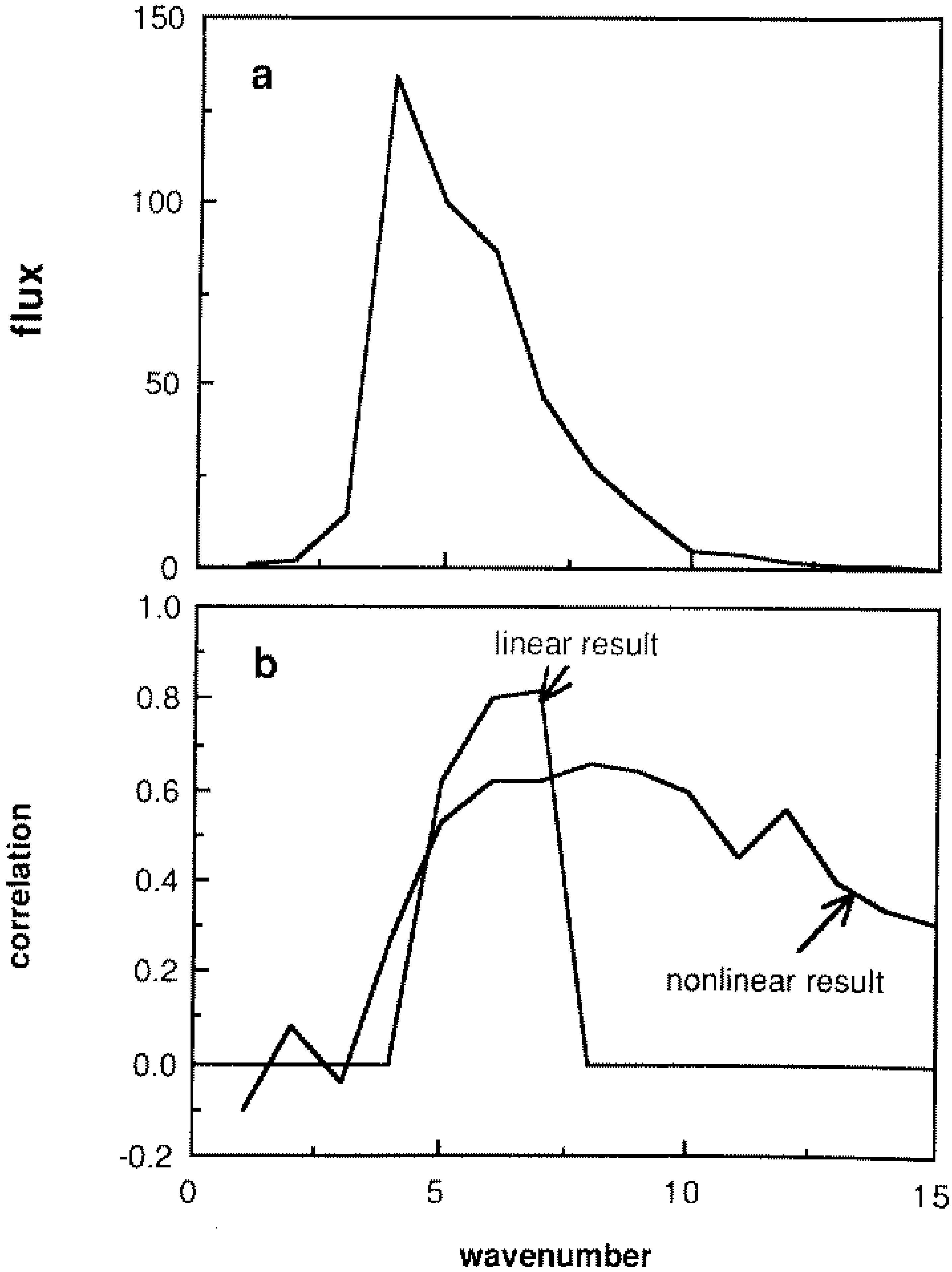

Figure 8. (a) Amplitude of potential vorticity flux as a function of wavenumber in F4 ($k_\beta = 6{\cdot}0$). The most energetic wavenumbers perform most of the transport, although they are not necessarily linearly unstable. (b) Correlation between temperature and meridional velocity, $\langle \tau\psi_x \rangle$, from the nonlinear integration and from the corresponding linear problem. In the linear problem the correlation is set to zero for those modes which are linearly stable.

(i) ad hoc nature of the parametrizations—the derivations are at best phenomenological;

(ii) evaluation of the magnitude of the transport coefficients—energetic arguments are rather arbitrary, especially in the light of the results of section 3, where the eddy amplitude is determined mainly by the forcing on the *zonal* flow, and in the presence of turbulent scrambling;

(iii) maintenance of momentum flux constraints—rather awkward manipulations and requirements on the transport coefficients are needed to maintain zero net momentum flux;

(iv) energetic consistency: a downgradient diffusion of potential vorticity will normally lead to energy nonconservation.

Regarding baroclinic adjustment, the argument in its disfavour is that nonlinear transfer obviates the need for it. Perhaps a more modest, although not qualitatively dissimilar, goal for a parametrization scheme is that of simulating subgrid-scale effects in a medium resolution (or large-eddy simulation) model. The advantages of scale separation disappear but the form of the parametrization may be less crucial. In two-dimensional and geostrophic turbulence the most sophisticated subgrid-scale schemes involve using a closure theory linked to an explicit simulation. In practice, especially for non-homogeneous flows, this will be too difficult to implement. In two-dimensional turbulence at medium to high resolution the term $-\nu\nabla^6\psi$ has been found an adequate enstrophy remover. An alternative—the anticipated potential vorticity method of Sadourny and Basdevant (1985)—replaces this term with $\theta J\{\psi, \nabla^n J(\psi, q)\}$. This maintains strict energy conservation while ensuring enstrophy dissipation. See also Vallis and Hua (1987). Here we shall examine the use of a simple downgradient flux of potential vorticity as an eddy parametrization for low and medium resolution models. Rather than try to derive constraints between the values in the model layers we use constant values for each. Our reason for this is that using a constant value for the transfer coefficients is simple, tractable, physically based (somewhat) and has been used, and probably will continue to be used, by many other workers. Thus, we parametrize the effects of subgrid-scale eddies by the term $K\nabla^2 q$ in each layer, where K is a constant. Figure 9 displays typical barotropic energy spectra so found. The value of the coefficient is tuned so that the energy in the energy-containing scales is similar to that found in a higher resolution model. The significant effect of using such a scheme is that the energy spectra are too shallow. A conventional parametrization ($\nabla^4\zeta$) is quantitatively better at this resolution, although it produces a dissipation range at higher wavenumbers. The parametrization is unable to remove enstrophy at the smaller scales efficiently enough. If the value for the eddy diffusion coefficient is increased to try to compensate for this, then the simulation

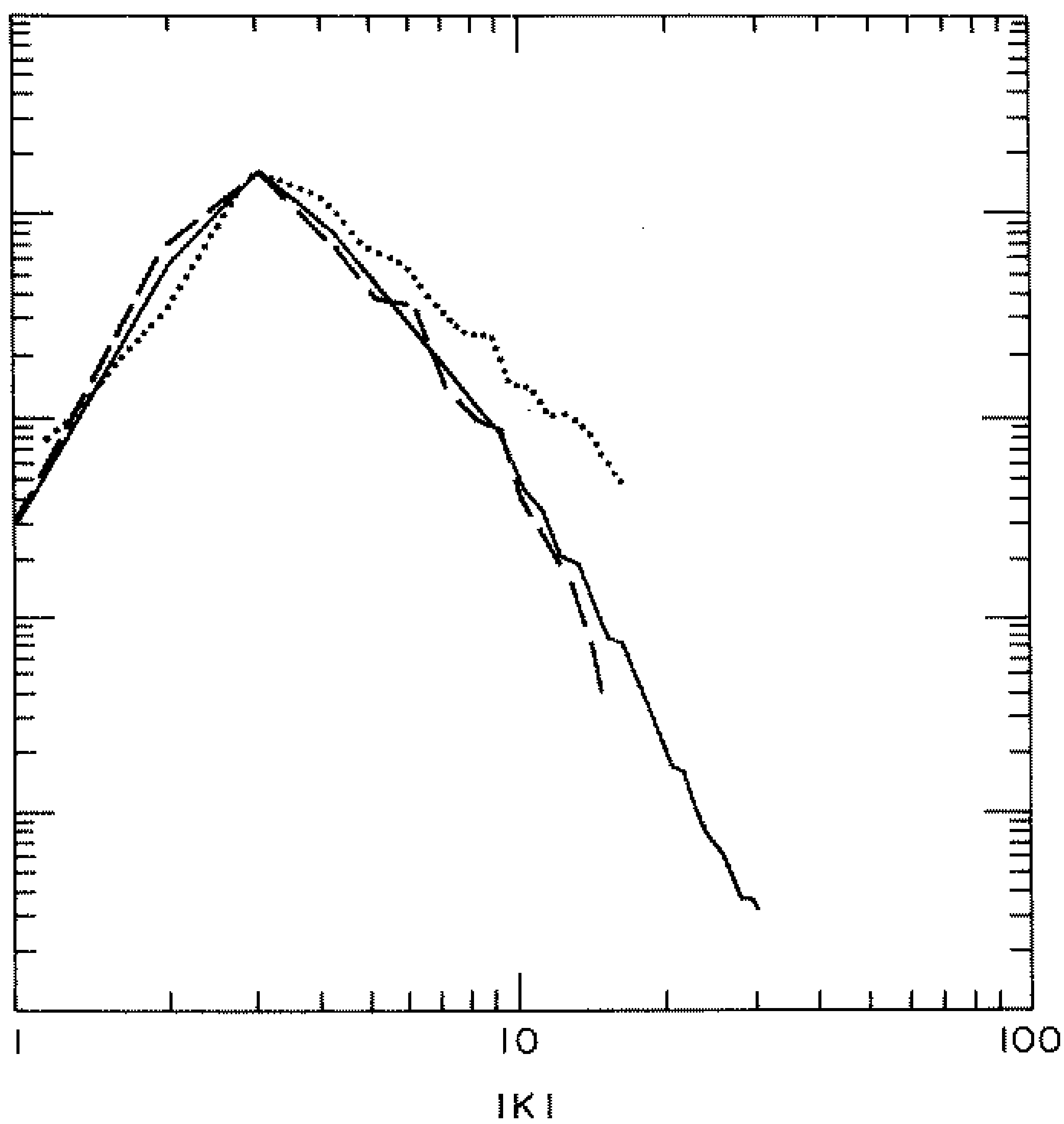

Figure 9. Energy spectra using potential vorticity diffusion as a subgrid-scale parametrizer (dotted line). The solid line is for a higher resolution model (for which the form of the subgrid-scale parametrization is much less important). The dashed line uses a $\nu\nabla^4\zeta$ scheme. This produces a dissipative range, but is quantitatively closer to the high resolution run than the potential vorticity diffusion.

becomes too viscous and too lacking in energy at all scales. The spectral slope is actually rather sensitive to the value used, and can then become too steep. In summary, the parametrization is not scale selective enough to be used in low to medium resolution models. Making the parametrization more scale selective by increasing the power of the diffusion operator would of course help (the parametrization then becomes like a conventional enstrophy remover) but potential vorticity is no longer diffused, and we have resigned ourselves to an ad hoc scheme. Making the transport coefficient proportional to the local time-averaged instability will do no good in this regard since for a homogeneous model this is the same everywhere, and there is no reason why a scheme should work for an inhomogeneous model and not for a homogeneous one. Making the coefficient proportional to the local, instantaneous energy of the resolved flow may work, but seems rather contrived since we do not know how much mean energy is converted to eddy energy. If, as here, the cut-off scale is in the enstrophy inertial range (or, strictly, in the inertial range of the higher resolution model we seek to mimic) then no energy flows into the unresolved scales and any dissipative parametrization is unrealistic. However, for very low resolution models, such as in White and Green (1982) where the effects of all the transient baroclinic fluxes are parametrized, the scheme qualitatively correctly reproduces the dissipative effects of the transient eddies on the asymmetric and zonal mean flow. This may be in part because the eddies being parametrized are energy containing and do act to dissipate energy in the mean flow, unlike eddies in the inertial range.

5. Discussion and conclusions

In this paper we have looked at some transport properties associated with large-scale, nonlinear, geostrophic flow. Such flow is a characteristic of flow regimes found in mid-latitude tropospheric flow and gyre-scale flow in the ocean. Although our parameters and boundary conditions are perhaps somewhat more appropriate for atmospheric flow, some of our general results must hold for the ocean. Before discussing the results, a comment about doubly-periodic flow is in order.

In modelling atmospheric flow, channel models are a common enough substitute for spherical geometry. We have chosen an even greater simplification, namely doubly-periodic flow, for a number of reasons. The first is just computational efficiency—a doubly periodic domain allows a simple basis set of eigenfunctions and hence allows higher resolution to be reached. Secondly, it allows for homogeneous statistics, properties we have used in deriving relations between the transfer coefficients and the fluxes in section 4. Thirdly, it allows an explicit scale separation between the mean flow and the eddies, the most favourable condition for transport theories. A disadvantage may be seen in that the boundary conditions are unrealistic, first in not allowing an obvious formulation for the mean flow variation, and second in being unphysical (for try to consider the conservation of potential vorticity on a particle as it leaves the domain at the top ($y = 1$, say) and reenters at the bottom ($y = 0$), apparently feeling immediately a lower value of the planetary vorticity). The first objection is overcome by using the formulation for rate of change of the mean flow given in section 2 (and elsewhere). Similarly the second point is really not so bad, and certainly does not corrupt the dynamics: Note that the dynamics only recognizes the planetary vorticity gradient, β, which is constant everywhere. Also, a physical way to think of the flow is that of a very large field of eddies, existing on an infinite β plane possibly with an imposed large-scale shear. If we only allow eddies much smaller than the scale of the mean shear, we can, purely for computational convenience, choose to represent the eddy field as periodic flow. Provided the width of our box is larger than the energy-containing scales of the eddies, the statistics of the dynamics within the box will be the same as if we were truly on an infinite β plane. In particular the flux of quantities across the box is not affected by the periodicity.

The simplicity of the geometry and, more importantly, the scale separation between eddy and mean flow, allowed a physical derivation to be given for the rate of change of the mean flow under the action of the eddies. The derivation is not based on purely energetic considerations, nor is it valid only for barotropic flow (although in the absence of topography it maintains Galilean invariance). Under the influence of this mean flow, the eddy flow is unstable, and fluxes of heat and potential vorticity are induced. We looked at the equilibration properties of the flow, both for the fully nonlinear model and arbitrarily truncated models. We found that if the eddy field is highly truncated, so that only one wavenumber mode is allowed, baroclinic adjustment indeed does occur. It also occurs if all eddy modes are present, but only wave–mean-flow, and not wave–wave, interactions are allowed. In that case, although many eddy modes may initially be excited, ultimately all but one die down and the mean flow equilibrates at the lowest shear which will give baroclinic instability, and only one wavenumber is excited.

If more than one wavenumber is allowed to represent the asymmetric field, baroclinic adjustment does not generally occur, if the forcing is sufficiently strong. For then multiple asymmetric modes will be excited and baroclinic adjustment need not operate. Indeed, if only one triad of eddy modes is allowed, the mean flow is able to equilibrate at a supercritical level. Similarly, if all the meridional modes are allowed at a given zonal wavenumber, supercritical equilibration can occur. Baroclinic adjustment will normally not occur if other asymmetric modes in the eddy field (i.e. in addition to the single most unstable mode) are present. This is because equilibration is then a nonlinear process, and the energy-containing eddies transfer their energy in an energy cascade to larger scales. Whereas the backtransfer of energy to larger scales in our models, and in the atmosphere and ocean, is hardly inertial (surface drag, for example, is important) the gross characteristics of the transfer will approximate transfer in an energy inertial range in the sense that the energy transfer will be spectrally fairly local, because the energy transfer is associated with the straining of the velocity field by its own shear (Kraichnan 1967). (This is somewhat different from the situation at higher wavenumbers where the energy spectra fall steeply and transfer in the enstrophy inertial range is relatively nonlocal, and each octave contributes equally to the mean straining.) Thus, the eddies need not equilibrate by further interaction with the mean flow, unless no other option is open to them. Energy will *cascade* back to the zonal flow. A question not addressed in this paper is the strength of the forcing in the real atmosphere and ocean. The many simplifying assumptions make direct parameter comparison impossible, although the essential dynamics is faithfully mirrored in so far as the mid-latitude atmosphere is quasi-geostrophic and mainly driven by a meridional temperature gradient. (The magnitudes of the various parameters chosen (friction, domain size, etc.) certainly affect the quantitative results. However, they do not affect the nature of the conclusions expressed here.) In the real atmosphere and ocean it is probably an open question as to how strong the forcing is. If it is sufficiently weak, then baroclinic adjustment could still hold. However, the observed presence of several energetic asymmetric modes would argue against this.

The transfer, or flux, of potential vorticity is frequently assumed to be downgradient, and parametrized using Austauch coefficients. In our models the former assumption holds. The latter, namely the Austauch parametrization, is seen to be somewhat unsatisfactory. First, the magnitude of the coefficients cannot quantitatively be given by a measure of the instability of the mean flow, because the flow can equilibrate at highly supercritical values, and so much of the available potential energy of the mean flow is not used. This objection could be met if a known fraction of the mean APE were used to drive the eddies, but the author is not aware of such knowledge. Second, the parametrization does not conserve energy, which makes it unsuitable if the maximum wavenumber of the parametrized model is in the inertial range, although not necessarily unsuitable if the maximum wavenumber is in the energy-containing range. Third, because potential vorticity is not passive, prior specification of the transfer coefficients requires, in order that momentum be conserved, a rather arbitrary specification of their vertical structure. (On the other hand, this argument could be reversed and said to give otherwise unknown information about the coefficients. See, for example, White and Green (1982).) Fourth, using it as a subgrid-scale parametrization in medium resolution models is not recommended because the eddy viscosity is then constant at high wavenumbers, whereas closure calculations and explicit calculation show that it should increase rapidly to a cusp at the truncation wavenumber (Kraichnan 1976). This could be overcome if the coefficients are not local in space (but instead have some local energy weighting, say). However, this procedure would be undeniably ad hoc, especially in the light of the first point above. Of course, if the real atmosphere or ocean is only slightly supercritical then setting the transport coefficient proportional to the supercriticality or to the available potential energy in the mean flow will be an acceptable approximation. This question cannot be answered with the model used here, again because the model is too unrealistic for direct parameter checks. Finally, none of the above arguments in any way implies that potential vorticity is not transported downgradient. Indeed there are arguments based on examination of the enstrophy budget in the presence of dissipation that show that in many cases it must be.

ACKNOWLEDGMENTS

This work has been funded by the National Science Foundation under grant ATM84-12044 and by DARPA/ONR under the University Initiative Program No. N0014-86-K-0758.

REFERENCES

Carnevale, G. F. and Frederiksen, J. S. 1987 Nonlinear stability and statistical mechanics of flow over topography. *J. Fluid Mech.*, **175**, 157–181

Green, J. S. A. 1970 Transfer properties of the large-scale eddies and the general circulation of the atmosphere. *Q. J. R. Meteorol. Soc.*, **96**, 157–185

Harrison, D. E. 1978 On the diffusion parameterization of mesoscale eddy effects from a numerical ocean experiment. *J. Phys. Oceanogr.*, **8**, 913–918

Hart, J. E. 1979a Finite amplitude baroclinic instability. *Ann. Rev. Fluid Mech.*, **11**, 147–172

1979b Barotropic quasi-geostrophic flow over anisotropic mountains. *J. Atmos. Sci.*, **36**, 1736–1746

Held, I. M. 1975 Momentum transport by quasi-geostrophic eddies. *ibid.*, **32**, 1494–1497

Kraichnan, R. H. 1967 Inertial ranges in two-dimensional turbulence. *Phys. Fluid.*, **10**, 1417–1423

1976 Eddy viscosity in two and three dimensions. *J. Atmos. Sci.*, **33**, 1521–1536

Loesch, A. Z. 1974 Resonant interactions between unstable and neutral baroclinic waves. Parts I and II. *ibid.*, **31**, 1177–1217

Marshall, J. C. 1981 On the parameterization of geostrophic eddies in the ocean. *J. Phys. Oceanogr.*, **11**, 257–271

Marshall, J. C. and Shutts, G. 1981 A note on rotational and divergent eddy fluxes. *ibid.*, **11,** 1677–1680

Pedlosky, J. 1970 Finite amplitude baroclinic waves. *J. Atmos. Sci.*, **27,** 15–30

Rhines, P. and Young, W. R. 1982 Homogenization of potential vorticity in planetary gyres. *J. Fluid Mech.*, **122,** 347–367

Sadourny, R. and Basdevant, C. 1985 Parameterization of subgrid scale barotropic and baroclinic eddies in quasi-geostrophic models: Anticipated vorticity method. *J. Atmos. Sci.*, **42,** 1353–1363

Salmon, R. S. 1980 Baroclinic instability and geostrophic turbulence. *Geophys. and Astrophys. Fluid Dynam.*, **15,** 167–211

Stone, P. H. 1978 Baroclinic adjustment. *J. Atmos. Sci.*, **35,** 561–571

Vallis, G. K. 1985 On the spectral integration of the quasi-geostrophic equations for doubly-periodic and channel flow. *ibid.*, **42,** 95–99

Vallis, G. K. and Hua, B-L. 1987 Eddy diffusivity of the anticipated potential vorticity method. *J. Atmos. Sci.* (in press)

Veronis, G. 1981 Dynamics of large-scale ocean circulation. In *Evolution of physical oceanography.* Eds. B. A. Warren and C. Wunsch. MIT Press

White, A. A. and Green, J. S. A. 1982 A nonlinear atmospheric long wave model incorporating parametrizations of transient baroclinic eddies. *Q. J. R. Meteorol. Soc.*, **108,** 55–85

1984 Transfer coefficient eddy flux parametrizations in a simple model of the zonally averaged circulation. *ibid.*, **110,** 1035–1052