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Conceptual Models of El Niño and the Southern Oscillation

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We present a few simple models which are intended to encapsulate some of the basic mechanisms of the El Niño/Southern Oscillation phenomenon. We consider one- and two-dimensional, continuous and low order models, with and without external stochastic forcing. In the low order models, even in the absence of stochastic forcing, chaos and aperiodic ENSO events can occur. This behavior is, however, rather sensitive to the choice of parameters and to the precise difference formulation used. Some of the detailed behavior of very low order models can also be unrealistic. However, the presence of multiple solutions is robust and insensitive to the differencing assumptions. One notable result of these models is that El Niño events partially phase-locked to the seasonal cycle can be produced both by by variations in trade-wind intensity and by imposed annual cycles in the temperature forcing. A continuous model which reduces to a chaotic low-order model in the limit of very coarse finite differencing is presented. Multiple, analytically derivable solutions exist which, however, are stable to infinitesimal perturbations. Larger perturbations or stochastic forcing can cause irregular oscillations between the two stationary states and El Niño like events, even in the absence of equatorial waves. Adding gravity waves produces somewhat more oscillatory behavior. Sustained oscillations leading to El Niño-like events are easy to produce with the addition of a seasonal cycle and some random noise. El Niño events are then found as occasional amplifications of the seasonal cycle. The difference between El Niño "events" and irregular amplifications of the seasonal cycle is then rather arbitrary. A common feature in all models is an oscillation between two equilibria due to an instability, either linear or to finite size perturbations, of the coupled ocean atmosphere system. The robustness of this suggests it may be a feature of the real system. The timing within events is governed by the detailed dynamics allowed in the particular model, but oscillatory behavior is readily obtained simply by allowing sufficiently strong, but not too strong, coupling between model atmosphere and ocean, plus perhaps some noise.

1. INTRODUCTION

Interest in the El Niño/Southern Oscillation (ENSO) phenomenon has been high for some time. Indeed, much of a recent edition of the *Journal of Geophysical Research* (volume 92, issue C13) was devoted to observations and consequences of it. One obvious reason for this is the importance of the events both for tropical oceanography itself and for local fisheries, as well as for the possible influence of the tropical sea surface temperature field on the global climate. Another reason is that it appears that it is now becoming possible to explain, at least in outline, the causes of the phenomenon. The coupling between atmosphere and ocean induces a positive feedback, which coupled with simple ideas of equatorial oceanic dynamics (stemming from the use of the shallow water equations) enables explanations to be offered which have a striking simplicity. (Of course, such explanations are not necessarily right.) Thus various models have been rather successful in explaining some of the grosser features of the events [*McWilliams and Gent*, 1978; *Cane and Zebiak*, 1985; *McCreary and Anderson*, 1984; *Philander et al.*, 1984; *Lau*, 1981; *Vallis*, 1986; *Schopf and Suarez*, 1987]. The simplest of all these models is perhaps that of *Vallis* [1986], henceforth V86. Analytic stability arguments are available, and the model can be studied in detail. However, the very simplicity of the model brings drawbacks. The model is so simple that it clearly cannot be compared directly with observations. Further, some of its behavior is unrealistic (see section 3 below). The question arises, is the behavior of such a model completely artifactual, or do the principal mechanisms exist in more complex models? It is the purpose of this paper to try to use the simple ideas and principal mechanisms thought important for the ENSO cycle in models as simple as possible, but without producing artifactual behavior.

Let us first briefly review the main features of the ENSO phenomenon. We shall often refer to El Niño "events", (and even El Niños) but without prejudice. Thus such a terminology will not preclude the possibility that these are merely manifestations of a sustained oscillation.

1. El Niño is the occurrence of an anomalously warm pool of water in the eastern equatorial Pacific Ocean. The event lasts for a few months.
2. Concurrent with the ocean warming, an atmospheric event occurs, namely a notable weakening of the trade winds. As one indicator, the sea level pressure difference between Darwin and Tahiti is correlated with the oceanic events.
3. The event occurs aperiodically, with intervals of between 2 and 11 years, but typically they are 2 to 5 years apart. There have been nine or so events since 1945, when reliable records began, with large events in 1957, 1965, 1972, and 1982. There is evidence for El Niño events for over 400 years [*Quinn et al.*, 1987].
4. The event is phase locked to the seasonal cycle, normally reaching its maximum amplitude around Christmas time. However, large variations can occur, notably in 1982–1983.
5. Similar events, if they occur at all, are much weaker in the Atlantic and Indian oceans.

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The large scale of the event is indicative that it can be explained with "low-order" models, and this is the justification for much of the modeling effort. One of the first simple models was that of *McWilliams and Gent* [1978]. They did not find highly oscillatory behavior in realistic parameter regions. As will be shown below, such results are quite sensitive to the particular finite differencing used, and their lack of nonlinear oscillations should not be taken as a sign of their absence in the real system, any more than the results of V86 necessarily imply the contrary. More realistic ocean dynamics were incorporated by *McCreary and Anderson* [1984]. Although their ocean models had rather more physics (advection, gravity waves, etc.) it seems that their results were dominated by a simple feedback mechanism between model ocean and atmosphere. Still more realistic models, especially for the atmospheric component, were presented by *Cane and Zebiak* [1985] and by *Schopf and Suarez* [1987]. Cane and Zebiak's ocean is a single-layer, linear reduced gravity model, and their atmosphere a simple tropical circulation model. Only deviations from the mean state of either atmosphere or ocean are predicted. For a reasonable range of parameters, nonlinear oscillations and aperiodic El Niños ensue. No explicit stochastic forcing is needed in some parameter ranges to produce aperiodic behavior, although in other regimes, regular oscillations ensue. Many of the results are usefully explained by the mechanism described by *Philander et al.* [1984] as unstable oscillations in a coupled, interacting air-sea system.

A somewhat different type of model was presented by *Lau* [1981], in which stochastic forcing is an inherent component. It is worth commenting on the role of stochasticity. In general if a high order or continuous fluid dynamical system is truncated to low order, a number of courses of action are possible with respect to the neglected modes. First, they can be ignored (as in *Vallis* [1986], and elsewhere). Second, an "eddy viscosity" of one form or another can be invoked. This normally takes the form of an enhanced friction, representing the damping effects of the neglected modes. Third, a stochastic forcing can be added, representing the apparently random activity of the subgrid scale modes [e.g., *Egger*, 1981]. Finally, the most consistent approach of all is to use both eddy damping and stochastic forcing in a consistent way such that the appropriate invariants, such as energy, are conserved in the inviscid limits. This is a formidable task, which cannot be carried out except in the simplest of fluid systems. The point is that it is not inconsistent to use random forcing in a large-scale system if only the gravest few modes are being simulated. The presence of randomness does not necessarily mean that the overall dynamics is not dominated by large-scale interactions, nor does it mean that such forcing is in any sense necessarily an intrinsic part of the system, other than insofar as the effect of the subgrid scale modes must be parameterized.

Before describing the models to be used here in more detail, a further comment is in order pertaining to simple models of ENSO in general. Some criticism has been laid at the foot of such models for being overly simplistic. Such criticism is fair if the models are demonstrably unphysical. Still, I believe that the models can be useful in laying bare the essential physics, or shaving away the unessential. They perhaps play the same role to ENSO theory as, in atmospheric science, do low-order models to blocking [e.g., *Roads*, 1980], or energy balance models to climate studies [*Ghil*, 1976]. Nobody suggests that these models are to be taken too literally, nor can their equations be derived systematically without unjustified assumptions from the equations of motion. Yet the mechanisms they propose (for example, resonance in blocking, ice-albedo feedback in climate change) may yet have some validity in the real world, and in any case the physics can be made much clearer through the simple models. With regard to atmospheric variability, other studies with more complicated models have modified conclusions as necessary and added more or less important mechanisms. For example, resonance of the stationary Rossby waves with topographic forcing now seems less important than previously indicated using low-order models, but the idea of topographic instability has become universal. The models presented here are still very much at the same level as the energy balance models, or the low-order blocking models. It is hoped that they nevertheless are of some interest.

2. SIMPLE MODELS OF ENSO

2.1. *A low order chaotic model*

We shall first describe a very simple model of ENSO, similar to that of *Vallis* [1986], and compare and contrast it to similar models. We imagine that all of the essential dynamics are confined to a zonal plane at the equator. We further suppose that two temperatures suffice to describe the thermal state of the upper ocean, namely, T_w and T_e, the near-surface temperatures in the wetern and eastern ocean. The deep ocean is supposed to have a constant temperature $\overline{T}$, which is unaffected by motion on the time scales of concern to us (namely, months to years). We further suppose that the temperature is affected by a surface current u (see Figure 1). Now, if the motion in this plane is divergence-free and incompressible, then the equation of continuity may be written

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

Let the fields u and w be such that they are represented by scalar values U and W at $x = 0$, $z = 1/2$, and $z = 0$, $x = \pm 1/2$, respectively. A centered difference formulation of this, given the above crude differencing, would be

$$\frac{U}{\Delta x} - \frac{W}{\Delta z} = 0 \qquad (1)$$

where we have assumed the scalar field u is zero at the zonal boundary, and w is zero at the ocean surface. The use of the continuity equation will make the system in Figure 1 topologically equivalent to a stretched ocean in which the deep ocean is placed alongside the upper surface. Thus we imagine another grid point a distance Δx to the west of T_w (equidistant with T_e) at which the temperature is held at $\overline{T}$. Indeed, one could physically imagine that the water advected into the equatorial channel comes not from the deep ocean but from higher latitudes. Now consider for a moment the continuous equation for temperature advection. We write this as

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z} = 0$$

From Figure 1, it is clear that a centered finite difference formulation to this is

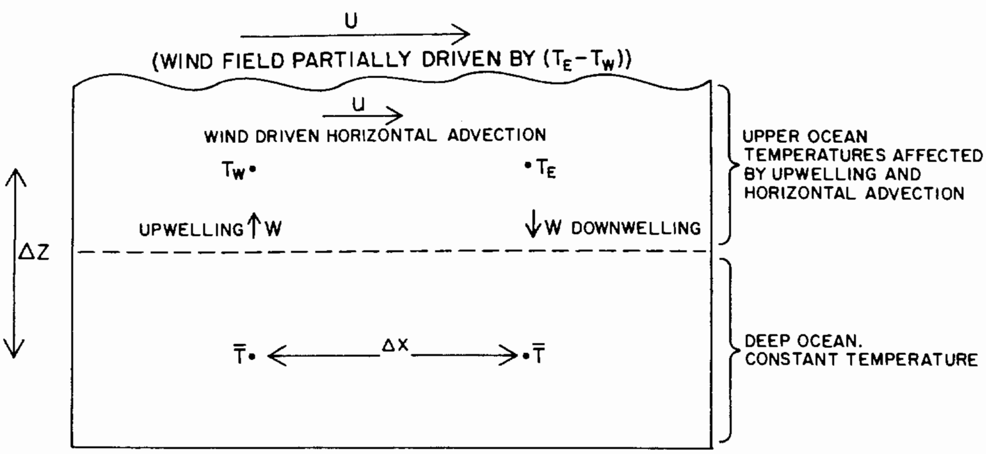

Fig. 1. Schematic diagram of two-point model [after Vallis, 1986].

$$\frac{dT_w}{dt} = \frac{U}{2\Delta x}(\overline{T} - T_e)$$

$$\frac{dT_e}{dt} = \frac{U}{2\Delta x}(T_w - \overline{T})$$

Forcing and damping terms are obviously important in the equatorial ocean. Let T^* be the temperature to which the ocean would relax in the absence of motion, for simplicity keeping this the same for the eastern and western ocean. Then exchange of sensible and latent heat with the overlying atmosphere may be crudely parameterized by a Newtonian damping with a time scale A^{-1}. The above equations become

$$\frac{dT_w}{dt} = \frac{U}{2\Delta x}(\overline{T} - T_e) - A(T_w - T^*) \qquad (2)$$

$$\frac{dT_e}{dt} = \frac{U}{2\Delta x}(T_w - \overline{T}) - A(T_e - T^*) \qquad (3)$$

These equations are not closed until we have an equation for U. Probably the simplest assumption to make about the atmosphere is that the surface wind is forced by a quantity proportional to the temperature difference across the ocean, in addition to the constant easterly wind provided by the trades. Thus we write

$$\frac{du_a}{dt} = B'(T_e - T_w) - C'(u_a - u^*) \qquad (4)$$

where u_a is the atmospheric surface wind, C' is a damping time scale and u^* represents the mean effects of the trade winds. Here u^* will be negative if the trades are easterlies. If the time scales on which T_e and T_w are changing are of the order of a few months, and the damping time scale C'^{-1} is much shorter than this, then (4) will always be in approximate equilibrium. In that case we have

$$u_a = \frac{B'}{C'}(T_e - T_w) + u^* \qquad (5)$$

The surface wind in turn provides a stress on the upper surface of the ocean, which we parameterize as a body force thus:

$$\frac{dU}{dt} = Du_a - CU \qquad (6)$$

where C^{-1} is a frictional time scale for the upper ocean. In this equation we have ignored the effects of any pressure gradient in the ocean which may tend to absorb the body force and counter the wind stress. This effect will be discussed in section 4, but for the time being we shall ignore it. If we assume the atmosphere to be always in equilibrium (i.e., we use (5) instead of (4)) then (6) may be replaced by

$$\frac{dU}{dt} = B(T_e - T_w) - C(U - U^*) \qquad (7)$$

B is given by DB'/C and $U^* = Du^*/C$. The above equations, i.e., (2), (3), and either (4) and (6) or (7), form a closed set for the variables T_w, T_e, and U and, if needed, u_a. The equations (2), (3), and (7) are those derived by *Vallis* [1986]. Before looking at other, equally plausible (or implausible) models, we shall examine the stability properties and explicit behavior of these in numerical simulations. Tractable analytic stability results prove to be possible only in the special case of $U* = 0$ (since the resulting cubic factorizes in this case). However, no generality is lost by setting $\overline{T} = 0$, since this merely defines the zero level of the temperature. With these simplifications, we shall write the equations as

$$\frac{dU}{dt} = B(T_e - T_w) - CU \qquad (8)$$

$$\frac{dT_w}{dt} = -\frac{U}{2\Delta x}T_e - A(T_w - T^*) \qquad (9)$$

$$\frac{dT_e}{dt} = \frac{U}{2\Delta x}T_w - A(T_e - T^*) \qquad (10)$$

The equations simplify with the following substitutions: $\hat{t} = At$, $\hat{u} = U/(2A\Delta x)$, $\hat{T}_w = T_w/T^*$, and $\hat{T}_e = T_e/T^*$
This yields

$$\frac{d\hat{u}}{d\hat{t}} = \frac{\hat{B}}{2}(\hat{T}_e - \hat{T}_w) - \hat{C}\hat{u}$$

$$\frac{d\hat{T}_w}{d\hat{t}} = -\hat{u}\hat{T}_e - (\hat{T}_w - 1)$$

$$\frac{d\hat{T}_e}{d\hat{t}} = \hat{u}\hat{T}_w - (\hat{T}_e - 1)$$

where $\hat{B} = T^* B/(\Delta x A^2)$, and $\hat{C} = C/A$. Thus there are just two governing parameters determining the system's behavior. The equations can now be put in a form very reminiscent of the Lorenz equations by forming the sum and difference of the two temperature equations. Defining $y = (T_e - T_w)/2$ and $z = (T_e + T_w)/2$, we obtain

$$\frac{d\hat{u}}{d\hat{t}} = \hat{B}y - \hat{C}u$$

$$\frac{dy}{d\hat{t}} = uz - y \tag{11}$$

and

$$\frac{dz}{d\hat{t}} = -uy - (z-1)$$

The Lorenz equations may be written (substituting u for the usual x)

$$\frac{du}{dt} = -\sigma u + \sigma y$$

$$\frac{dy}{dt} = ru - uz - y \tag{12}$$

$$\frac{dz}{dt} = uy - bz$$

The only essential differences between these equations and the set (11) are that in the Lorenz equations the perturbation temperature y and the mean temperature z decay with time scales differing by the factor b, which is an aspect ratio. (Make the substitution $z' = z - 1$ in (11) for clarity.) The structure of the two sets is the same. This correspondence is broken, however, if the term U^* (the trade wind effect) in the original set is kept. This gives an east-west asymmetry to the set which cannot be removed. The similarity between the sets is more than formal. The Lorenz equations describe a single cell of Benard convection, heated from below. Although the flow is normally thought of as driven by a vertical temperature gradient, it is actually the horizontal temperature difference between the ascending and descending branches which produces a corresponding density difference, which in turn provides a buoyancy force to drive the flow. In our set the vertical temperature gradient is represented by $\overline{T} - T^*$, which will normally be negative. The system is similarly driven by a horizontal temperature gradient, which (envisioning basically a large atmospheric convection cell) produces a wind to drive the ocean. In our ocean, large-scale overturning occurs with the surface warmer than the deep ocean; this can occur because the ocean is being mechanically driven by the wind stress. Only the overlying atmosphere need be thermally driven. In the Lorenz model the Prandtl number, σ (= ν/κ, is the ratio of viscous dissipation to thermal dissipation. The analogous parameter here is $\hat{C}$, the ratio of time scales of decay of sea surface temperature anomalies to a frictional time scale. The aspect ratio is unity in my model. The control parameter governing the strength of the air-sea interaction and the vertical temperature difference is $\hat{B}$ in my model and r in Lorenz's. Much analysis has of course been done on the Lorenz equations, so we shall only briefly outline how the stability properties of our set may be obtained.

The steady solutions of the set are obtained by setting the left-hand sides to zero and solving the resulting cubic. This can readily be done. There is one trivial solution of no motion in which $u = y = z = 0$, and a pair of solutions for which

$$\hat{u}^2 = \frac{\hat{B}}{\hat{C}}(1 - \frac{\hat{C}}{\hat{B}})$$

$$y^2 = \frac{\hat{C}}{\hat{B}}(1 - \frac{\hat{C}}{\hat{B}})$$

$$z = \frac{\hat{C}}{\hat{B}}$$

We see that for $\hat{C}/\hat{B} \geq 1$, only the trivial solution is realizable. Now $\hat{B}$ is a measure of the strength of the feedback between ocean temperature and surface wind. If this is small or nonexistent, only the resting solution is possible. This evidently makes physical sense. The system undergoes a pitchfork bifurcation at $\hat{C}/\hat{B} = 1$, as two new solutions appear (and the existing one becomes unstable).

The stability properties of these solutions are found by substituting the steady solutions back into the original equations and linearizing. Searching for solutions of the form $e^{\sigma t}$, one obtains the following cubic equation:

$$\sigma^3 + \sigma^2(2 + C) + \sigma(C + B/C) + 2(B - C) = 0$$

A critical point occurs when the coefficient of σ^2 times the coefficient of σ equals the constant term. We thus find instability when the following inequality holds:

$$\hat{B} \geq \frac{(4 + \hat{C})\hat{C}^2}{C - 2}$$

For $\hat{B}$ less than this critical value (B_c) the eigenvalues are complex, but the real part is negative. Above B_c the real part is positive. Hence the system undergoes a Hopf bifurcation at this point. Given (perhaps a large assumption) that the above model equations represent in some sense the dynamics of large-scale equatorial air-sea interactions, what are appropriate values for the parameters Δx, T^*, B, and C? For the Pacific Ocean we choose Δx = 7500 km. T^* must be related to a typical temperature across the thermocline, say, 10° to 15° [*Levitus*, 1982]. The parameters C and A are harder to estimate, although only their ratio is dynamically important. Note that the model will always be stable if $C/A \leq 2$. However, one would certainly not expect this kind of detailed prediction to hold in a more complete model. Reasonable choices might be of the order of a few months for both. B is the parameter which is the hardest to estimate. Experiments with more realistic coupled atmosphere-ocean models are really the only possibility of coming up with quantitative estimates. Thus we shall use it as our control parameter and see how varying it changes the model behavior. If we choose a value for $B\Delta x$ of $2 \mathrm{m}^2 \mathrm{s}^{-2} \mathrm{C}^{-1}$, a frictional decay time scale of 1/2 month^{-1}, a temperature decay time scale of 1/6 month^{-1}, then $\hat{C}$ is 3 and $\hat{B}$ is 102, and we have instability. It is not my intention to defend these quantitative values vigorously, other than to say none are obviously wrong by more than a few factors, except perhaps for $\hat{B}$, whose value is not known. With these parameter values, chaotic behavior is obtained (Figure 2). Indeed the time series of u or y is the same as that obtained in an appropriate regime for the Lorenz attractor.

Of course, the time series in Figure 2 looks nothing like that of El Niño. This can be partially remedied by restoring the asymmetry between east and west by introducing a nonzero U^*. Figure 3 shows a time series with $U^* = -0.45$ ms^{-1}, and a dimensional time scale. El Niño-like events are clearly seen, rather more clearly than in the real world. The addition of a seasonal cycle does much to make the time series more respectably noisy. There are two parameters to potentially vary to give a seasonal cycle, namely, $U^*$and T^*. Varying U^* will obviously lead to the onset of the events being phase locked to the seasonal cycle: although the model becomes hard to mathematically analyze with nonzero U^*, it is clear that a large negative U^* leads to the model preferentially orbiting around one fixed point of the attractor. The larger the U^*, the harder it is to break away. With U^* varying, the model events preferentially occur when U^* is weak. However, evidence for strongly varying surface trade winds is arguable [e.g., *Gill*, 1982, pp. 461–462]. It is equally likely that the effective temperature forcing (in our model $T^* - \overline{T}$) varies seasonally. Now the stability parameter $\hat{B}$ is actually $B(T^*\text{-}\overline{T})/A^2$. Increasing the temperature contrast implies a more unstable system. Thus if T^* varies, the model will be more unstable (and more likely to produce an event) when T^* is high. Figure 4 shows that this is the case.

There are some unrealistic features of the model. Most noticeable is that the temperatures themselves, or the sum of the temperatures T_e and T_w, frequently become negative or, dimensionally, less than $\overline{T}$ (Figure 3). Since water is being advected from the deep ocean which is at $\overline{T}$, it seems paradoxical that this should occur within the model. The resolution to this lies in an examination of the difference scheme. The advection part of the equation for T_w is

$$\frac{dT_w}{dt} = \frac{U}{2\Delta x}(\overline{T} - T_e)$$

If U is positive, water is being brought up from the deep. If $\overline{T}$ is less than T_e, then T_w will fall, irrespective of its own value. Thus conceivably, T_w can fall below $\overline{T}$. Also, T_e will only be cooled if T_w is less than $\overline{T}$. One way to crudely overcome this effect is to use a formally less accurate upstream differencing scheme, as now described.

2.2. *An Upstream Model*

The model equations (2) are based on a centered differencing scheme. An alternative, not necessarily better or worse, is an upstream scheme. The advective part of the equation for T_w may obviously be written

$$\frac{dT_w}{dt} = \frac{U}{\Delta x}(\overline{T} - T_w) - A(T_w - T^*) \qquad U > 0 \qquad (13a)$$

$$\frac{dT_w}{dt} = \frac{U}{\Delta x}(T_w - T_e) - A(T_w - T^*) \qquad U < 0 \qquad (13b)$$

$$\frac{dT_e}{dt} = \frac{U}{\Delta x}(T_w - T_e) - A(T_e - T^*) \qquad U > 0 \qquad (14a)$$

$$\frac{dT_e}{dt} = \frac{U}{\Delta x}(T_e - \overline{T}) - A(T_e - T^*) \qquad U < 0 \qquad (14b)$$

These equations no longer allow the possibility of temperatures less than $\overline{T}$. The equation for u remains (8). In a similar manner to that used above, we can find the stationary solutions and then examine the stability properties of these solutions. The equations also have an east-west symmetry: reversing the sign of u and substituting T_w for T_e and T_e for T_w leads to an identical equation. Thus we need only consider one sign of U. Forming the sum and difference temperatures $y = (T_e - T_w)/2$ and $z = (T_e + T_w)/2$, we write the equations for $U > 0$:

$$\frac{dU}{dt} = by - cU$$

$$\frac{dy}{dt} = \frac{U}{2}(z - 3y) - y$$

$$\frac{dz}{dt} = -\frac{U}{2}(y + z) - (z - 1)$$

The stationary solutions are

$$y = -\frac{c}{b} + \sqrt{\frac{c}{2b}}$$

$$U = -1 + \sqrt{\frac{b}{2c}}$$

$$z = \frac{c}{b} + 3\sqrt{\frac{c}{2b}}$$

Note further that these solutions only exist for $c < 2b$ because we require $u > 0$. For $c > 2b$ the only solution is the trivial one of no motion.

A stability analysis is possible for this system also. The interesting result we find now is that both solutions are unconditionally stable. No nonlinear oscillations are now possible. Indeed numerical integrations confirm that the system will settle down to one of the two solution branches. Thus the chaotic nature of the solutions seems to be dependent on the truncation scheme. However, the fact that the solutions bifurcate into two branches, one branch with eastward currents and winds and warm temperatures in the east (a model El Niño state) and one with westward winds, is robust.

2.3. *Other Differencing Schemes*

A number of differencing schemes are of course possible. With a different oceanic aspect ratio, one could write

$$\frac{dT_w}{dt} = U(\overline{T} - \frac{T_w + T_e}{2})$$

and

$$\frac{dT_e}{dt} = -U(\overline{T} - \frac{T_w + T_e}{2})$$

but this leads to the trivial result $d/dt(T_w + T_e) = 0$. An equally plausible scheme is the following, for $U > 0$:

$$\frac{dT_w}{dt} = U(\overline{T} - \frac{T_w + T_e}{2})$$

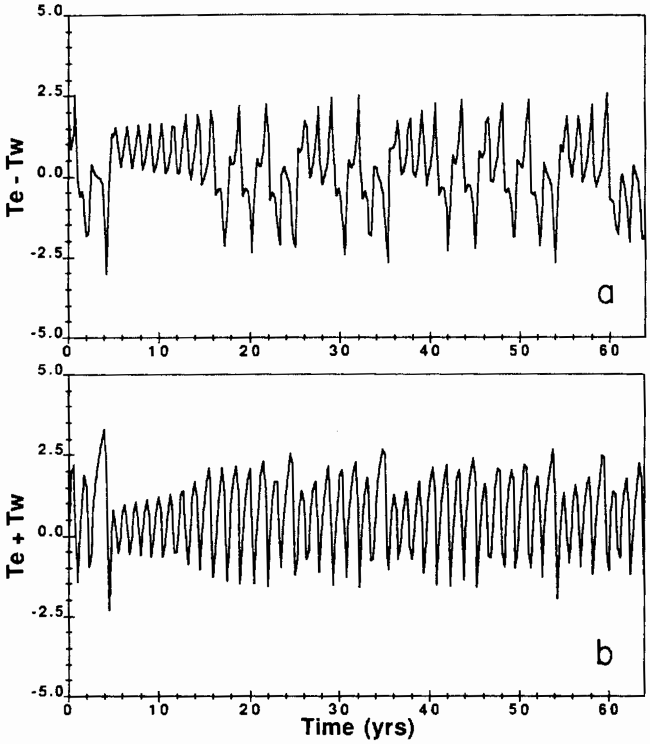

Fig. 2. Time series of two-point model in an unstable regime with $U^* = 0$. (*a*) Temperature difference $T_e - T_w$. (*b*) Temperature sum $T_e + T_e$.

and

$$\frac{dT_e}{dt} = U(T_w - T_e)$$

with obvious changes for $U < 0$. This is the scheme used by *McWilliams and Gent* [1978]. Using this scheme in our order 3 system leads only to steady solutions and no oscillations. Although their system is different in other ways, notably in having a pressure parameterization, this does point to one reason why the behavior in their model is rather different from that of V86: namely, no highly oscillatory solutions are possible with their differencing scheme.

In summary, then, we can describe our experience with low-order models as follows. Using a centered differencing scheme and reasonably realistic parameters, a very simple model will produce self-sustained oscillations and El Niño-like behavior, producing events every few years or so. The behavior is governed by two parameters, a Prandtl number and a control parameter B. B is physically dependent on the strength of the ocean-atmosphere feedback and the vertical temperature difference across the upper ocean. If this parameter is too small, no oscillations are possible. Further, varying either this parameter or U^*, governing the strength of the trades, produces events which are partially phase locked to the seasonal cycle. However, using other, equally plausible, differencing schemes shows that the chaotic behavior is not universal. However, the production of a bifurcation in the solutions is maintained. Thus the simple presence of a feedback between ocean temperature and surface wind and thence surface current enables all models to exit in one of two states (or possibly three, counting the trivial one at rest). One of the states is like an El Niño state and one is a "normal state." If an imposed westward wind blows, then the east-west symmetry is broken, and the El Niño state has a smaller attractor basin.

The possible artifactual nature of low-order models is well recognized in turbulence theory. For example, *Curry* [1978] shows how the behavior of the Lorenz model is seriously modified by the addition of extra modes. On the other hand, in some circumstances the equations certainly do have direct physical relevance, as in the Malkus-Howard-Welander model [*Malkus*, 1972]. With these thoughts in mind, we shall now describe a continuous model similar to the low-order models described above. The model still contains no rotational dynamics and can only be derived heuristically from the equations of motion.

3. A Simple Continuous Model

Most of the above low-order schemes can be considered as abbreviations of the following partial differential equations:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = A(T^* - T) \qquad (15a)$$

$$\frac{du}{dt} = B\left[T(l) - T(0)\right] + C(u^* - u) \qquad (15b)$$

The equation is first order in x. Hence only one boundary condition is needed. Since information is propagated downstream only, we supply

$$T(0) = \overline{T} \qquad u > 0$$

$$T(l) = \overline{T} \qquad u < 0$$

We nondimensionalize by the following substitutions: $\hat{u} = u/LA$, $\hat{x} = x/l$, $\hat{t} = t^*/A$, $\hat{T} = T/T^*$ and set $\overline{T}$ to zero with no loss of generality. We then find

$$\frac{\partial \hat{T}}{\partial t} + \hat{u}\frac{\partial \hat{T}}{\partial x} = 1 - T$$

and

$$\frac{d\hat{u}}{dt} = \hat{B}\left[T(l) - T(0)\right] + \hat{C}(\hat{u}^* - \hat{u})$$

where $\hat{B} = BT^*/lA^2, \hat{C} = C/A$. The similarity to the finite difference models is obvious. Physically, we can imagine incompressible flow along a narrow tube of length l. Since the flow is incompressible, its velocity is uniform. The temperature of the fluid is determined by advection and exchange of heat through the walls of the tube, which we have parameterized as a Newtonian term on the right-hand side of (15). Since there is no diffusion, the only boundary condition needed is the upstream one, the temperature of the entering fluid. The velocity along the tube, we shall suppose is governed by the temperature difference across the flow, mimicking in a sense the ocean-atmosphere feedback.

The systematic derivation of (15) from the primitive equations governing equatorial dynamics cannot be defended vigorously. However, they have some similarity to the linearized one-dimensional shallow water equations used by *McCreary and Anderson* [1984] and others. These equations may be written

$$\frac{\partial u}{\partial t} - \beta y v + g\frac{\partial h}{\partial x} = \text{forcing}$$

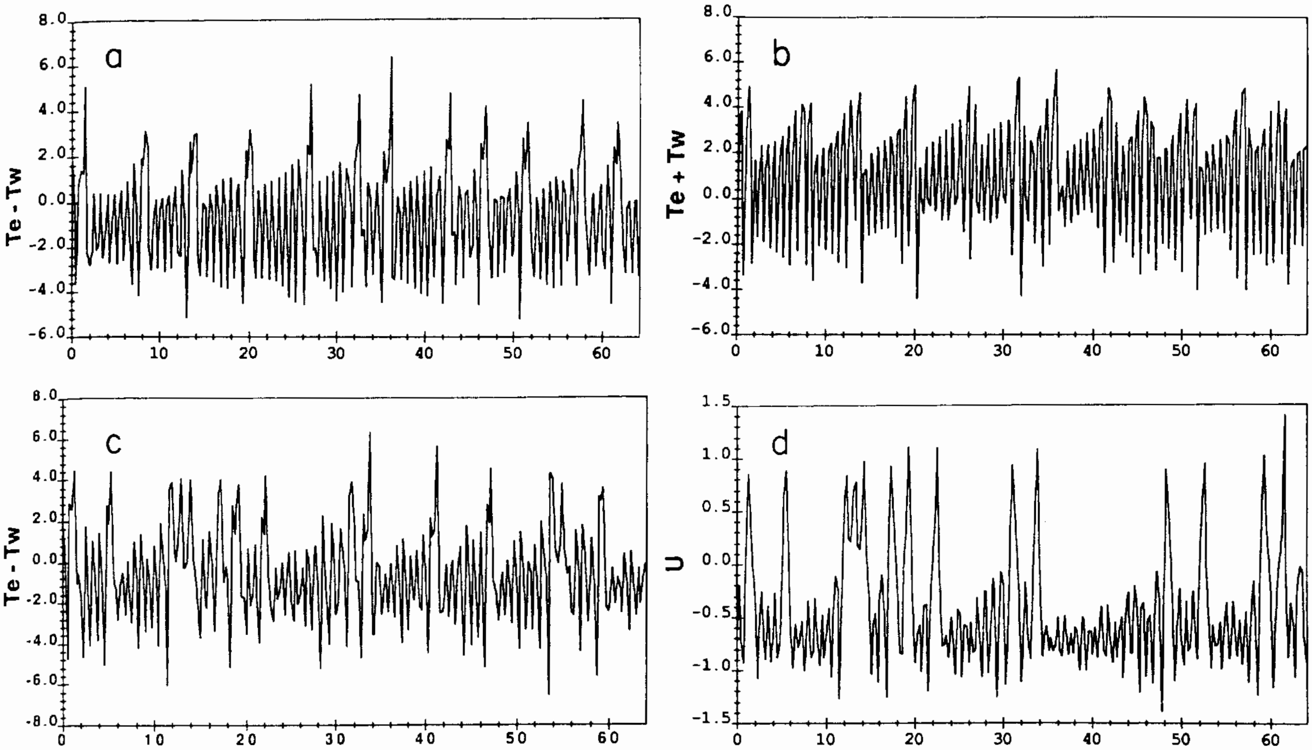

Fig. 3. Time series of various quantities for two-point model, with $U^* = -0.4$ ms^{-1}. (*a*) Temperature difference $T_e - T_w$. (*b*) Temperature sum $T_e + T_e$. (*c*) Temperature difference $T_e - T_w$ with a seasonal cycle imposed by varying the parameter T^*. (*d*) As for Figure 3*c* but for u.

$$\frac{\partial v}{\partial t} + \beta y v + g\frac{\partial h}{\partial y} = \text{forcing} \tag{16}$$

$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} + \frac{\partial hv}{\partial y} = 0$$

within a closed domain $0 < x < L$ and $-L_y < y < L_y$, say. Confining motion further to a zonal plane at the equator and assuming that the meridional velocity is much smaller than the zonal, then

$$\frac{\partial u}{\partial t} + g\frac{\partial h}{\partial x} = \text{forcing} \tag{17a}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} = 0 \tag{17b}$$

This is a consistent and a closed set, provided that at the boundaries, we specify the inflow height. The continuity equation is satisfied in (17*b*). McCreary and Anderson associated the height field with the surface temperature (since a deep thermocline leads to warm surface), and in turn parameterized the surface wind as a simple function (in fact a step function) of the h field. If we associate h with T then (17) is the same as (15), except for the varying pressure term in (17) and provided we choose the forcing term proportional to the difference in dynamic height across the domain.

We shall now present the analytic solutions of (15). Then we shall show that different finite difference formulations of (15) do lead to the simple models discussed in section 2. Then we will solve the partial differential equation numerically. In the next section the equations will be solved numerically with some additional stochastic forcing.

3.1. *Analytic Solutions*

For a given u we can straightforwardly write down the solution to (15). It is

$$T = 1 - exp(-x/u) \qquad u > 0$$

$$T = 1 - exp\big[(1-x)/u\big] \qquad u < 0$$

For $u > 0$, say, this states that the temperature (zero at $x = 0$) rises as x increases (due to heat exchange with the surroundings) asymptotically approaching its limiting value of unity. (If T^* is negative, the dimensional temper-

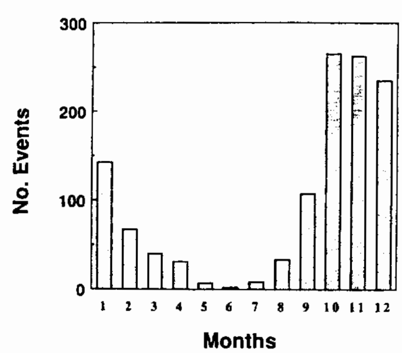

Fig. 4. Histogram of model event onset times (defined here by U becoming positive). Here a seasonal cycle was introduced by varying the parameter T^*, as for Figure 3.

ature falls to T^* as x increases.) For $u < 0$, the flow and temperature gradient are simply reversed.

The value of the flow u is given by solving the equation

$$u-u^* = \hat{B}/\hat{C}[T(1)-T(0)] = \hat{B}/\hat{C}[1-exp(-1/u)] \quad u > 0$$

or

$$u-u^* = -\hat{B}/\hat{C}[T(1)-T(0)] = \hat{B}/\hat{C}[1-exp(1/u)] \quad u < 0$$

This may easily be done graphically. An exact analytic expression is not available, but the solution may also be obtained iteratively. Note that two solutions again occur and that the solution of no motion no longer exists, even when u^* is zero. If u^* is zero, then the two solutions are symmetric. However, this is lost for nonzero u^*.

The stability of these solutions may also be obtained. The algebra is long and not immediately informative. The results are that there are no amplifying perturbations around the stationary solutions. That is, the solutions are stable fixed points.

3.2. *Differencing the Continuous Equations*

If the index representing the x coordinate is j, and considering only the advective part of (15) then a centered scheme is

$$\frac{dT(j)}{dt} = u[T(j-1) - T(j+1)]$$

Suppose there are only two active gridpoints, labeled 1 and 2. The boundary grid points are labeled 0 and J. Then at gridpoint 1 we have

$$\frac{dT(1)}{dt} = u[T(0) - T(2)]$$

and at 2

$$\frac{dT(2)}{dt} = u[T(1) - T(J)]$$

This is actually inconsistent with the continuous equations, since two boundary values must be specified (this holds for any centered scheme at any resolution). It has occurred because the differencing scheme has introduced some numerical diffusion, a second-order effect which requires two boundary conditions. If $T(J)$ and $T(0)$ are both specified as zero, the temperature equations become (9) and (10). Further, if in the equation for u the temperature difference across the domain is approximated by $T(2) - T(1)$, then the u equation is identical also to (8).

An upstream differencing may be preferable in that it does not require an extra unphysical boundary condition. We write

$$\frac{dT(j)}{dt} = u[T(j-1) - T(j)] \qquad u > 0$$

$$\frac{dT(j)}{dt} = u[T(j) - T(j+1)] \qquad u < 0$$

With two grid points, these lead to the upstream scheme discussed in section 2.

It is not surprising that we can recover the simple models discussed above. An advantage of the continuous model is that we can find analytic solutions and see if the numerics is relevant.

3.3. *Numerical Solutions*

We obtain numerical solutions to (15) and (16) by time stepping the equations with either a centered or an upstream scheme in the x direction, and no numerical diffusion. The upstream successfully reproduces the analytic solution and its stability properties. This is not the case for the centered scheme. With no explicit numerical diffusion (i.e., none other than that implicit in the differencing scheme), the model can apparently produce chaotic behavior. This is very similar to that produced by the centered model of section 2. Inspection of the numerical output reveals the presence of two-grid point oscillations. An explicit diffusivity can damp this, but if it is sufficiently large, then steady solutions prevail. The centered scheme is actually solving the continuous equations, plus a large nonlinear diffusion term which appears to be reproducing the chaotic behavior.

3.4. *An Explicit Atmosphere*

We can explicitly add an atmosphere as described in section 2. Thus (16) is replaced by the two equations:

$$\frac{du_a}{dt} = B'[(T(L) - T(0)] + C'(u_a^* - u) \qquad (18)$$

$$\frac{du_o}{dt} = Du_a - Cu_o \qquad (19)$$

In the two-point centered model, the solution remains chaotic. Although the number of degrees of freedom has been increased from 3 to 4, the qualitative behavior of the system is little altered when (realistically) $C \ll C'$. In the language of dynamical systems, the dimensionality of the attractor has not increased by much, and is probably still less than 3. Varying the parameter B again leads to stability or instability. In the continuous model it can be shown, after much algebra, that steady solutions exist and, as before, the solutions remain stable.

3.5. *Stochastic Forcing*

Stochastic forcing is a device for simulating the effects of the unresolved scales of motion and neglected dynamics on the resolved dynamics. For certain simple situations (generally homogeneous turbulence) the presence of such a term can more or less be derived from first principles. In those cases a renormalized viscosity should appear also. In geophysically interesting cases no recipe is available for determining its form or its magnitude, and one must proceed, if at all, on less secure physical intuition.

In the equatorial ocean dynamics, the following dynamics has been neglected in the models used above: rotational dynamics, including the effects of equatorial and Rossby waves; variations in pressure field giving equatorial Kelvin waves; smaller-scale interactions in the atmosphere, including cyclone activity and CISK-like phenomena; interactive effects with the rest of the atmospheric general circulation (e.g., Hadley cell variability). It seems likely that one could conceive a model incorporating Hadley cell interaction in a more or less ad hoc fashion, but we do not attempt this here. The rotational effects are crudely included by confining the motion to a zonal plane at the equator. The pressure field is necessary if the velocity field is allowed to spatially vary. It is responsible for maintaining a divergence-free, or incompressible, flow. By keeping u spatially fixed, we remove the need for that effect. That is to say, in the model above, suppose the velocity field is changed (say downwelling is prevented) on the western edge of the basin. Then the velocity field in the east must immediately change also (upwelling must halt). In a real fluid the changes are communicated through changes in the pressure gradient, which propagate as gravity waves.

The smaller-scale phenomena is impossible to model without a high-resolution coupled general circulation model. We shall use an ad hoc stochastic forcing, as did *Egger* [1981] and *Moritz and Sutera* [1981] (who added stochastic terms to the Lorenz model, among other things, broadening the range of irregular behavior). Thus (15) and (16) are replaced by

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = A(T^* - T) + ft(x,t) \qquad (20)$$

$$\frac{du}{dt} = B\left[T(L) - T(0)\right] + C(u^* - u) + fu(t) \qquad (21)$$

where $fu(t)$ and $ft(x,t)$ are random processes, not necessarily white. Since the solutions without stochasticity cannot analytically be written down, solving the Fokker Planck equation for the above set seems less straightforward than proceeding explicitly. Physically, we do not expect the subgrid-scale processes to be white, since many (e.g., tropical cyclones, variations in the trades) have time scales of weeks to months. A decorrelation time scale T may be introduced into a discrete time stepping scheme by writing

$$fu_i = \lambda fu_{i-1} + (1-\lambda^2)^{0.5} r_i A$$

where i is the time step, r_i is a random number uniformly distributed between -1 and +1, A determines the amplitude and λ the decorrelation time scale. If $fu(t)$ is to be white, then $\lambda = 0$.

For simplicity we shall set $ft(x,t) = 0$ and study stochasticity only in the velocity equation. All the effects we wish to demonstrate are reproduced in this way. We choose a decorrelation time scale of the order of 1 month, and an amplitude of the order of several meters per second. (Note that the amplitude and decorrelation time scale are related in their effects. If the decorrelation time scale is increased, the forcing has a much larger effect.) We also choose parameters within the unstable regime for the model of *Vallis* [1986] but ones stable (as are all parameters) for the continuous model. Figure 5 shows time series of u and of $\overline{T}_e - \overline{T}_w$, where $\overline{T}_e$ is the integrated temperature in the eastern half of the domain, and similarly for $\overline{T}_w$. For sufficiently strong, and not unrealistically so, stochastic forcing, the model is able to oscillate between normal and "El Niño" states. With a nonzero U^*, the model preferentially remains in the normal state, occasionally flipping to orbits surrounding the "El Niño" state. The velocity amplitude is rather large; this is due to the neglect of a dynamic height field, which will be remedied in the coming section.

The addition of stochastic temperature forcing adds only quantitative differences.

4. INCORPORATION OF DYNAMIC HEIGHT FIELD

In this section we explore the effects of gravity wave propagation in our system. We do this because gravity and Rossby wave propagation are considered by many to be a integral part of the oceanic part of the Southern Oscillation cycle. Additionally, the slope in the thermocline provides a pressure gradient limiting the advection. There will be two respects in which our model will remain unrealistic, both stemming from its one-dimensionality. First there can still be no Rossby wave dynamics. Second, gravity wave dynamics in the presence of a rotating planet leads to gravity waves propagating in one direction only (eastward at the equator), this condition stemming from the imposition of boundary conditions at infinity [e.g., *Gill*, 1982]. The dynamics we shall integrate is described by

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = A(h^* - h) \qquad (22a)$$

$$\frac{\partial u}{\partial t} + g\frac{\partial h}{\partial x} = B(\overline{T}_e - \overline{T}_w) + C(u^* - u) \qquad (22b)$$

Here, $\overline{T}_e$ and $\overline{T}_w$ are average temperatures in the eastern and western halves of the model ocean, and we shall suppose that this is direcly proportional to the average dynamic height difference. The one bow to linearity lies in the neglect of the velocity advection term; this is is any case small and may lead to undesirable shocks. Equation (20a) is a continuity equation for an upper layer thickness. To it we have added a diabatic term on the right-hand side, as in *Anderson and McCreary* [1985]. We do this in part because we are parameterizing temperature through the height field, noting the strong positive correlation between temperature and upper layer thickness [e.g., *Graham and White*, 1988]. Thus the term represents the exchange of heat with the atmosphere, and we suppose that when the temperature rises, the layer thickness increases. Note the similarity of this equation with (15). Equation (22b) likewise is similar to (15b), except for the addition of the pressure term $g\partial h/\partial x$. If this term is removed, and if u^* is not a function of x, then $\partial u/\partial x = 0$. The equations are then isomorphic with (15). For boundary conditions we set h = const (100 m) at either end. The feedback term (the first term on the right-hand side of (22b)) we use a simple linear function so that the feedback is proportional to the average difference between the height field in the eastern and western halves of the model ocean.

Numerical integrations of (22) in the absence of any external forcing lead to steady solutions, as for (15). The presence of a pressure parameterization leads to a smaller amplitude for the velocity field than in the previous model, because an eastward velocity produces a deeper thermocline in the east and an opposing pressure gradient. An external source of noise is again necessary to produce sustained oscillations and El Niño "events". The addition of a seasonal cycle is insufficient, leading only to regular oscillations with a period of exactly 1 year. Again, however, the addition of some random noise is sufficient to drive the system between two states. Figure 6 displays a typical time series of $\overline{u}$ and the average height difference between the east and west halves of the ocean. There is one significant point to be made here. The dominant period in these time series is the seasonal cycle; the height field rises and falls in symphony with this. Every few years, the random forcing is sufficient to greatly amplify the warming in the east and produce what might be called El Niño events. However, they are in many ways just amplifications of the natural seasonal cycle. Occasion-

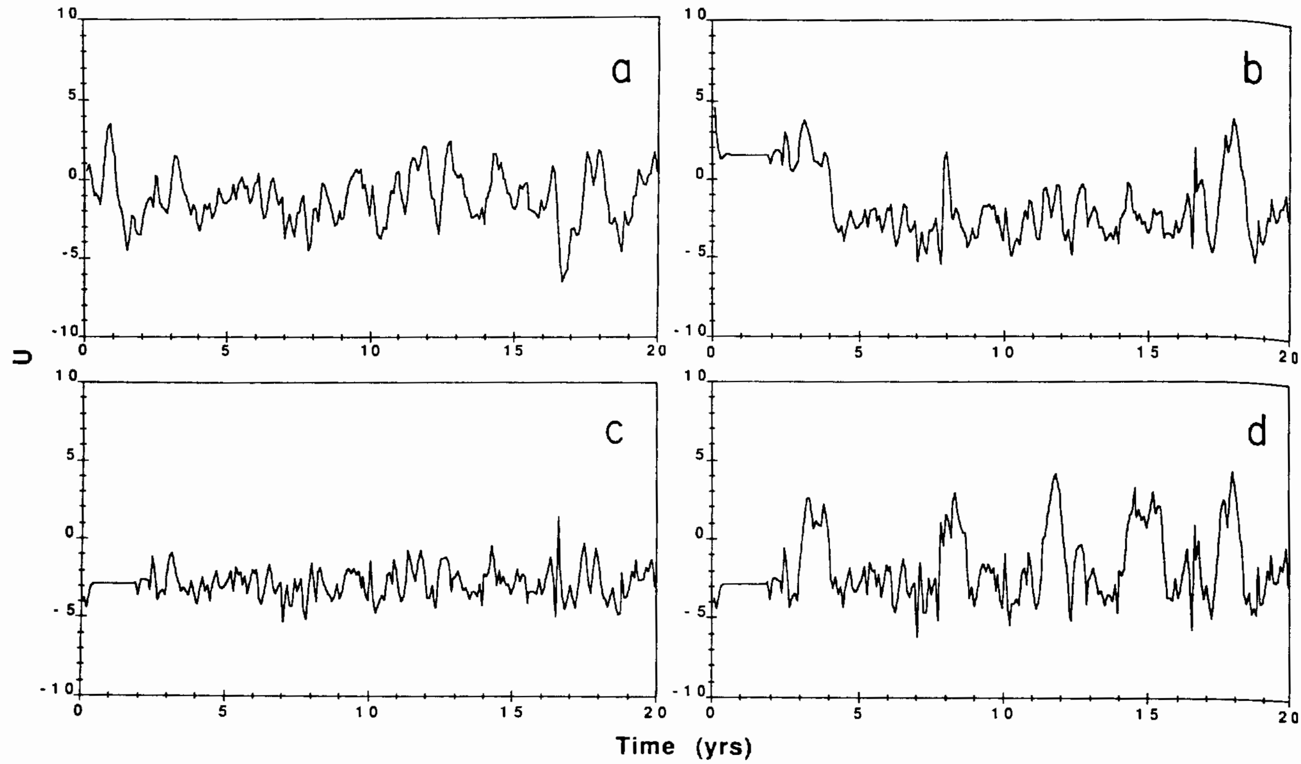

Fig. 5. Various time series of u for continuous model (15), showing effects of variations in strength of stochastic forcing. (*a*) No temperature feedback, $B = 0$, and random forcing governed by $\lambda = 0.99$ (approximately a 2-week decorrelation time scale, given a time step of 0.33 days) and $A = 30$. No El Niño like events occur. (*b*) As for Figure 5*a* but with temperature feedback ($B \neq 0$). (*c*) As for Figure 5*b* but with a shorter decorrelation time scale. The random forcing is now no longer strong enough to produce flips to the other solution state. (*d*) Same as Figure 5*c* but with stronger random forcing ($A = 40$). In Figures 5*b*, 5*c*, and 5*d* the random forcing was turned on after 2 years.

ally cold events in the east seem to be produced, although there is a marked asymmetry between warm and cold. This asymmetry is simply due to fact that the average wind is to the west (i.e., u^* is negative). Given this, the system resides preferentially in a "cold east/warm west" state, and the flips to a warm east state are more pronounced than amplifications of its normal state. The temperature-advection feedback is the crucial parameterization governing the cycle. If this is too weak, the velocity and height fields simply oscillate with the seasonal cycle, modified some by the noise (Figure 7). On the other hand, if the feedback is too strong, the system becomes locked in one state, and a much stronger perturbation would be necessary to cause it to produce oscillations large enough to be called El Niños.

El Niño events (or equivalently, amplifications of the seasonal cycle) are often preceded by cooling in the east. A time series of model dynamic height anomaly shows that the maximum dynamic height, centered normally in the mid-western ocean, moves west first before propagating east for the main event (Figure 7). This is not inconsistent with the observational studies of *Graham and White* [1988], although in the absence of model Rossby waves the westward propagation is simply due to gravity wave dynamics. The role of Rossby waves in this loop was recognized by *McCreary and Anderson* [1984].

5. DISCUSSION AND CONCLUSIONS

In this paper we have looked at a particular model, and variations around it, which contain a minimal set of "physics" pertaining to equatorial ocean-atmosphere dynamics and the El Niño/Southern Oscillation. The only physics they contain is a simple feedback between near-surface ocean temperatures and the atmospheric surface zonal wind, which subsequently provides a stress to the ocean and hence a current which advects the temperature field. It is possibly the barest set of physics which is of relevance and is obviously too simple to describe the richness of behavior possible in the equatorial system. Our purpose is not to do that, but to construct a canonical model of the system, showing that events like El Niño are virtually the natural consequences of that feedback and a mean westward forcing due to the trade winds. Certainly other processes, in particular, wave dynamics, will be important and may be crucial to do justice to the dynamics. Nevertheless, it has been demonstrated that this simple feedback is sufficient to produce noticeable events. The asymmetry between east and west is due only to the presence of mean easterly trade winds (a consequence of course, of rotational dynamics in the atmosphere), but other than this there is no dynamic asymmetry.

The simplest possible model containing a temperature-current loop was presented in V86. We have shown here that some of its behavior is unrealistic. One such feature is the production of surface temperatures lower than the deepwater temperatures, even though there is no possible physical source for such production. The artifactual nature of this is due to the use of a very coarse, centered in space finite difference scheme. If an upstream scheme (which is

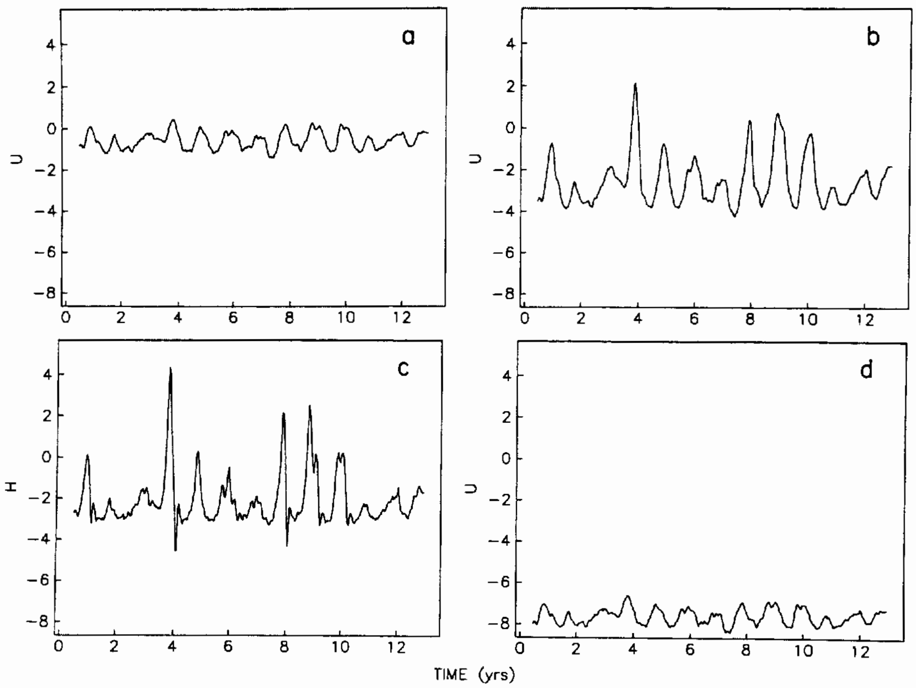

Fig. 6. Time series of x averaged zonal velocity ($\overline{u}$) and mean height difference between east and west ocean ($\overline{h}_e - \overline{h}_w$) for (20), with seasonal cycle in U^* and stochastic forcing, illustrating effects of changing feedback parameter B: (*a*) $\overline{u}$ for $B = 0$, (*b*) as for Figure 6*a* but $B = 6$, (*c*) as for Figure 6*b* but $\overline{h}_e - \overline{h}_w$, and (*d*) $\overline{u}$ for $B = 12$. The random number sequence is the same for all experiments.

not necessarily any more accurate) is used to remove this, the chaotic behavior also disappears. In other words, the bifurcation sequence leading to chaos is sensitive to the differencing scheme. (This is likely the cause of the rather different behavior found in V86 and by *McWilliams and Gent* [1978]. The two parameters governing the behavior of these models are the Prandtl number and a control parameter B. The first is the ratio of a frictional decay time scale to a temperature anonaly decay time scale. If this is too large, then the model is always stable. The second parameter governs the strength of the temperature-current feedback, as well as the vertical temperature contrast (between a below thermocline temperature and a relaxation temperature, representative of a near-surface air temperature). If this parameter is too small (and some would argue that it should be zero, that the ocean really does not affect the atmospheric winds), then the feedback is cut off and the instability dies.

A continuous model for which analytic solutions are available (and probably stable) was presented. In common with the low-order models, two nontrivial solutions exist, one with higher temperatures to the west and a westward current, and conversely. The addition of stochastic forcing provides finite amplitude perturbations which drive the model between its two states. If the mean effects of the trades (i.e., a mean westward wind forcing) are incorporated, the attractor basin of the warm-west state is larger than the warm-east state. The addition of stochastic forcing is now necessary to provide finite amplitude perturbations which drive the model between its two states. Most of the time the model lies near its "non El Niño" state, occasionally transiting to produce a warming event in the east. The transit time to the event is short compared with its residence in either state. It is governed by an advection time scale. Although a stochastic forcing is now necessary to produce the oscillations, the mechanism of the transition is just the feedback alluded to above. The point is that in the presence of finite size perturbations, neither the "normal" nor El Niño states need be unstable in order that the system may oscillate between the two states. The governing dynamics, even in the stochastic case, is still a large-scale air-sea interaction. Contrariwise, if the large-scale states are themselves unstable to infinitesimal perturbations, then no-smaller scale phenomena are needed, as in V86 and *Cane and Zebiak* [1985].

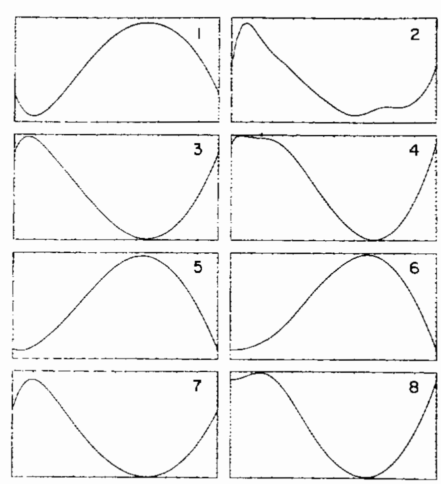

Fig. 7. Plots of successive values of the height anomaly, at quarter year intervals, through a model El Niño beginning at year 3 of Figure 6*b* or 6*c*.

An important point is that wave dynamics are not included this model, yet the time scales produced by the model are reasonably realistic. More specifically, the time interval between events is determined by the strength of the stochastic forcing and/or by the instability due to the temperature-current feedback. Reasonable parameter values can give intervals of the order of a few years. The transition between states is rapid compared with this; it is governed by advective and decay time scales (which are in turn governed also by the stability parameters). A parcel moving at 75 cm s^{-1} takes about 150 days to cross a 10,000-km basin, a reasonable, if rather slow, onset time scale. It is likely, however, that the parameters governing the transition time scale are seriously modified by wave dynamics, as found by *Schopf and Suarez* [1987], even if the numerical value is similar.

The addition of gravity wave dynamics and a seasonal cycle produces a model with rather oscillatory dynamics. In this model, El Niño events are seen as amplifications of a basic seasonal cycle. These amplifications may be triggered by any random noise, and the positive feedback between temperature (height) field and the velocity causes the model to produce large "events." Whether one chooses to call these phenomena events or oscillations seems rather irrelevant.

In none of the models presented here is there any explicit cutoff mechanism; that is to say, there is no new physical phenomenon (for example, off-equatorial Rossby waves) which is triggerred at the end of the "cycle" and which causes the event the subside. There is no need for one. In the chaotic models the El Niño state is unstable, and the system quickly returns to its usual state. In the stochastic models, external perturbations quickly knock the model from its perch. In both cases the maximum amplitude of the El Niño state is determined by the fact that the eastern ocean sea surface temperature cannot reach a higher temperature than exchange of heat with the atmosphere allows.

Determining if the real large-scale system is or is not linearly unstable to infinitesimal perturbations can probably only be decided by very complex models. It seems unlikely that it is very unstable, with growth of the order of a few months, because of the rarity of El Niño events and the apparent absence of other large oscillations in the system. Thus whereas in the mid-latitude atmosphere it is a meaningless question to ask what is the particular perturbation which brought about a particular cyclone since the theoretical steady state never exists, it may be sensible to look for appropriate perturbations for the equatorial system. If multiple equilibrium does exist, then oscillations between the two or more steady solutions can be excited by any number of perturbations. Indeed, it is not inconsistent that mid-latitude influences, transmitted for example, through variations in the intensity of the Hadley circulation, could "trigger" El Niño events. If the large-scale system is unstable, then self-sustained chaotic oscillations are a possibility. However, in the presence of an undeniably stochastic atmosphere (a consequence of course, of chaos in the large-scale atmosphere) chaos due simply to large scale equatorial ocean-atmosphere dynamics is not necessary, and the point may be moot.

We note that the chaotic nature of the system cannot be ruled out (or confirmed) merely by noting its apparent predictability on time scales of the order of a year or so. Chaotic systems in general, whereas certainly not displaying long-term predictability, certainly do not necessarily lack short-term predictability. Indeed, the short-term predictability of the system is likely to be higher if the system is governed in the main by large-scale dynamics rather than by stochastic forcing of much smaller space and time scales.

The precise importance of small-scale stochasticity can conceivably be answered by trying to determine the correlation dimension [*Grassberger and Proccaccia*, 1983] of the system, and there is some indication it may be small [*Hense*, 1986]. Given a very long time series, it is possible to determine the effective number of degrees of freedom of the system. If this is few, then small-scale processes are presumably not important. The time series of reliable data may not be long enough to determine this accurately. Further, it is also the case that one would need to filter any real data, spatially and temporally, to obtain meaningful answers since otherwise one would have a great deal of essentially stochastic noise ("essentially" because it would seem stochastic to the analyzer, although of course, it too is the result of a deterministic system). The stochasticity (the motion of cumulus clouds, for example) would fill out the dimension of the attractor almost to that of the entire phase space. It is not clear what effect filtering would have. Very long integrations of a realistic numerical model seem a possible alternative.

Further understanding of the equatorial ocean-atmosphere system will obviously also come from more realistic models. However, it will also be necessary to continue using simple sets of equations, derived where possible from the full set, on which analytic or very simple numeric work is possible. Simple models suggest important mechanisms and physical processes. Comparison of such models with the results from elaborate coupled ocean-atmosphere models, which themselves can be compared with data, will lead to more comprehension of the relevant dynamics.

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