

Jet Interaction and the Influence of a Minimum Phase Speed Bound on the Propagation of Eddies

AMANDA K. O'ROURKE AND GEOFFREY K. VALLIS

Princeton University, Princeton, New Jersey

(Manuscript received 6 November 2012, in final form 19 March 2013)

ABSTRACT

The feedback between planetary-scale eddies and analogs of the midlatitude eddy-driven jet and the subtropical jet is investigated in a barotropic β -plane model. In the model the subtropical jet is generated by a relaxation process and the eddy-driven jet by an imposed wavemaker. The authors propose a minimum zonal phase speed bound in addition to the established upper bound, where the zonal phase speed of waves must be less than that of the zonal mean zonal flow. Cospectral analysis of eddy momentum flux convergence shows that eddy activity is generally restricted by these phase speed bounds.

The wavenumber-dependent minimum phase speed represents a turning line for meridionally propagating waves. The authors vary the separation distance between the relaxation and stirring regions and find that a sustained, double-jet state is achieved when either a critical or turning latitude forms in the interjet region. The interjet turning latitude filters eddies by zonal wavenumber such that shorter waves tend to be reflected off of the relaxed jet and are confined to the eddy-driven jet. The interjet region is transparent to long waves that act to blend the jets and may be associated with barotropic instability. The eddy-driven and relaxed jets tend to merge owing to the propagation of these long waves through the relaxed jet waveguide.

AU1

1. Introduction

The maintenance and variability of the zonal mean zonal flow in the midlatitude upper troposphere can be linked to the eddy momentum fluxes of planetary-scale Rossby waves (Limpasuvan and Hartmann 2000). The eddy momentum flux associated with these waves is strongly tied to their meridional propagation through the upper troposphere. The meridional propagation of eddies is highly sensitive to the structure of the background flow—namely, the subtropical and eddy-driven jets. These two jets are dynamically distinct and are regions of enhanced meridional potential vorticity (PV) gradients through which eddies preferentially propagate, reflect, and break.

The zonal mean circulation of the midlatitude troposphere is characterized by two dynamically distinct mechanisms for the maintenance of zonal mean zonal winds—namely, angular momentum transport associated with the Hadley circulation (Schneider 1977; Held and Hou 1980) and the meridional convergence of eddy

momentum flux in the region of the storm tracks (Dickinson 1969; Thompson 1980). These mechanisms lead to production of the subtropical jet and the eddy-driven jet, respectively (Lee and Kim 2003); see Vallis (2006) for further discussion.

The eddy-driven and subtropical jets are not often separate entities in Earth's atmosphere, however. Though the zonal mean zonal wind in the Northern Hemisphere typically has only one westerly wind maxima, longitudinally localized regions often reveal a splitting of the jets and a hint at the two underlying mechanisms for jet maintenance. In the North Atlantic, for example, the splitting of the jet into distinct eddy-driven and subtropical components has been associated with the positive phase of the North Atlantic Oscillation (Vallis and Gerber 2008). Additionally, the Southern Hemisphere winter zonal average tends to have two zonal wind maxima indicative of subtropical and eddy-driven jets (Gallego et al. 2005).

The role of eddies in the maintenance and variability of these two jets is unsurprisingly complex. Eddies generated within the storm tracks propagate meridionally as Rossby waves that are highly sensitive to the background potential vorticity gradient. In the case of jets, the meridional curvature of the zonal mean zonal

Corresponding author address: Geoffrey Vallis, Princeton University, 300 Forrester Road, Princeton, NJ 08540.
E-mail: gkv@princeton.edu

wind at the center of westerly jets, such as the eddy-driven and subtropical jets, are regions of enhanced potential vorticity gradients known as Rossby waveguides (Harlander et al. 2000). From a barotropic perspective, waves propagate meridionally through these waveguides and set up a flux of eddy momentum from the region of wave breaking toward the region of eddy generation following the Eliassen–Palm (EP) flux formulation of wave–mean flow interaction.

The extent to which eddies propagate meridionally through the eddy-driven and subtropical jets, and thus the region of wave breaking, depends strongly on the zonal wavenumber and zonal phase speed of the wave (Randel and Held 1991). Meridional propagation can be inhibited by both the presence of critical latitudes, where the zonal phase speed of the wave matches that of the background zonal flow, and by turning latitudes where the meridional wavelength tends to zero owing to a weakening of the background potential vorticity gradient (Hoskins and Ambrizzi 1993).

Although the eddies themselves are generated in the region of the eddy-driven jet, the subtropical jet may be an important region for wave breaking and reflection (Magnusdottir and Walker 2000) and thus it is necessary to consider the interaction of two distinct jet types when discussing the propagation patterns of midlatitude eddies. Depending on the latitude at which waves break, the westerlies between the subtropical jet and the eddy generation region could be weakened, accentuating a two-jet structure, or the two jets could appear to merge with no distinct interjet region.

The type of wave breaking has sometimes been associated with changes in the phase of the annular modes (Limpasuvan and Hartmann 2000) and the related phenomenon of the North Atlantic Oscillation (Benedict et al. 2004). The meridional position of the combined eddy-driven and subtropical jets can be influenced by the type of wave breaking such that it can be shifted either poleward or equatorward of its mean position depending on the direction of the eddy momentum fluxes (Orlanski 2003). Rossby waves are observed to break either in a cyclonic or anticyclonic fashion within the midlatitudes from either equatorward or poleward propagation of waves generated within the storm tracks. Specifically, Orlanski (2003) find that preferential anticyclonic wave breaking can be attributed to the meridional asymmetries in the effective β arising from spherical effects. In the absence of spherical effects, is it still possible to create asymmetric wave breaking by changing just the zonal mean zonal wind?

Baroclinic instability within the storm tracks may not be the only source of eddies in the midlatitudes. There is some evidence to suggest that barotropic instability due

to horizontal shear of the eddy-driven and subtropical jets may be of interest to the dynamics of the zonal mean flow (Simmons et al. 1983; Kidston and Vallis 2010), but questions remain about both the importance and signature of barotropic instability. For example, how might the propagation pattern of eddies generated by barotropic instability differ from that generated by baroclinic instability? What consequences might that have on the merger and separation of the eddy-driven and subtropical jets?

It has been found that the interaction of eddies and the subtropical and eddy-driven jets varies depending on the zonal wavelength and zonal phase speed of the eddies. Son and Lee (2005) found that fast, high-frequency waves tend to drive a separation of the eddy-driven and subtropical jets while slow, low-frequency waves tend to merge the two jets in statistically steady state. In the context of climate change and the observed poleward migration of the midlatitude jets, Chen et al. (2007) note that changes in the eddy wave speed can drive a change in the location of the eddy-driven jet through a meridional movement of the location of wave breaking. For the same background flow, the results of Son and Lee (2005) suggest that waves of different phase speeds have fundamentally different interactions with the background flow. How, then, does the structure of the background flow, particularly the relative locations of the eddy-driven and subtropical jets, influence the propagation of eddies of varying phase speeds?

Additionally, Orlanski (2003) notes that the position of the eddy-driven jet can be influenced by cyclonic and anticyclonic wave breaking. Notably, Orlanski (2003) finds that anticyclonic (cyclonic) wave breaking can shift the eddy-driven jet poleward (equatorward). The meridional direction of eddy propagation determines the location of wave breaking on either the cyclonic or anticyclonic flank of the jet. How might the structure of the background flow lead to an asymmetry in wave propagation and breaking across the eddy-driven jet?

We consider here the interaction of the dynamically distinct eddy-driven and subtropical jets in a barotropic, β -plane model. Our approach draws heavily on the quasi-linear theory of wave propagation and thus depends on there being a meaningful partition between the zonal mean state and waves and/or eddies. To the extent that this assumption on scale separation holds, we propose new formulation of the refractive index to determine the limits of meridional eddy propagation in section 2. The model itself is described in section 3. We consider whether these jets show a similar tendency toward merger as observed in more complex models such as that of Son and Lee (2005). We will also determine whether this simple model can produce asymmetric wave

breaking—namely, either preferential cyclonic or anti-cyclonic wave breaking—due only to the presence of a subtropical jet waveguide and not to changes in the effective β due to spherical geometry as found in Orlanski (2003). The properties of jet merger and asymmetry in wave breaking are then discussed in section 4.

2. Linear theory and diagnostics

The character of eddy momentum flux and the horizontal propagation of Rossby waves through a background shear flow are investigated using cospectral analysis of the eddy momentum flux convergence. This method decomposes the eddy momentum flux convergence from the physical coordinates of longitude and time to zonal wavenumber and zonal phase speed (Hayashi 1971; Randel and Held 1991). Given that Rossby waves in a zonally symmetric flow will propagate meridionally away from the source of instability while conserving their zonal wavenumber and zonal phase speed, this coordinate system is useful in determining the limits to which waves may propagate in latitude.

Limits of wave propagation are described as either turning or critical lines and are latitudes at which the meridional group velocity of the wave packet tends to zero. To the extent that forcing and dissipation are small and that the flow can usefully be separated into a zonal mean and perturbation state, the limits of wave propagation can be obtained from the Rossby wave dispersion relation. For a single layer on a β plane, the potential vorticity equation is given by

$$\frac{\partial}{\partial t}(\nabla^2 \psi' - k_d^2 \psi') + \bar{U} \frac{\partial \nabla^2 \psi'}{\partial x} + \frac{\partial \psi'}{\partial x}(\beta - \bar{U}_{yy}) = 0, \quad (1)$$

where ψ' is the perturbation streamfunction of the velocity field, k_d^2 is the square of the deformation wavenumber, β is constant, and $\bar{U} = -\bar{\psi}_y$ is the zonal mean zonal velocity.

If the perturbation streamfunction is assumed to be wavelike in time and longitude with a meridionally varying amplitude,

$$\psi' = \Psi(y)e^{i(kx - \omega t)}, \quad (2)$$

we can then obtain the Rayleigh equation for waves propagating on a slowly varying, meridionally sheared flow (e.g., Vallis 2006, XX–XX):

$$\Psi_{yy} + \left(\frac{\beta - \bar{U}_{yy} + k_d^2 \bar{U}}{\bar{U} - c} - k^2 - k_d^2 \right) \Psi = 0. \quad (3)$$

We then define the refractive index as

$$n^2 = \frac{\beta - \bar{U}_{yy} + k_d^2 \bar{U}}{\bar{U} - c} - k^2 - k_d^2. \quad (4)$$

Solutions of the amplitude function $\Psi(y)$ in (3) are then wavelike when values of n^2 are bounded between $0 < n^2 < \infty$.

In regions where n^2 is negative, the waves are evanescent. The bounds on n^2 can then be translated into bounds on the zonal phase speed c . Given $\bar{U}$, c is bounded by

$$\bar{U} - \frac{\beta - \bar{U}_{yy} + k_d^2 \bar{U}}{k^2 + k_d^2} < c < \bar{U}, \quad (5)$$

where k is the zonal wavenumber and where β is assumed constant. Here the minimum and maximum phase speeds are

$$c_{\min}(y) = \bar{U} - \frac{\beta - \bar{U}_{yy} + k_d^2 \bar{U}}{k^2 + k_d^2} \quad \text{and} \quad c_{\max}(y) = \bar{U}. \quad (6)$$

Note that the limits in (5) are a function for the minimum and maximum phase speed at a given latitude rather than for the absolute bounds on the maximum and minimum phase speeds over the entire domain as discussed in Pedlosky (1986). The latitudinal dependence of (5) will be necessary to determine the local propagation characteristics of eddies in a meridionally sheared flow.

The behavior of eddies near the bounds given in (6) differs significantly. At the upper bound $c_{\max}$ the refractive index tends to infinity and we obtain wave breaking from linear theory. This is the typical situation addressed by critical line theory in which the zonal phase speed of the wave approaches that of the zonal mean zonal flow, as illustrated, for example, in the cospectra diagnostic of eddy momentum flux convergence (Randel and Held 1991).

Waves have a fundamentally different behavior at the lower limit of their phase speeds. The zero refractive index limit is one in which the meridional wavelength of the wave becomes infinite and the amplitude of the wave tends to infinity (Harlander 2002; Hoskins and Ambrizzi 1993; Yang and Hoskins 1996). From the perspective of Rossby wave ray tracing, wave packets approaching $n^2 = 0$ will be refracted toward regions of larger n^2 . Latitudes where n^2 approaches zero are known as turning latitudes or turning lines as waves tend to turn away from these regions.

The minimum phase speed due to the presence of turning lines may be usefully illustrated using the cospectra diagnostic of Randel and Held (1991). Unlike the upper bound on phase speed, the minimum phase

speed is highly sensitive to both the background potential vorticity gradient and to the wavenumber of the wave. For instance, long waves may attain a wider range of phase speeds than short waves for the same background potential vorticity gradient and thus may propagate meridionally farther than short waves. A large potential vorticity gradient, such as those found within the waveguides themselves where $U_{yy} < 0$, will have a wider range of possible phase speeds as compared to regions with a more homogeneous potential vorticity structure.

From the perspective of propagation alone, both critical and turning lines of a sheared flow denote regions of the flow where the meridional group velocity tends to zero. For our purposes, both critical and turning lines will be meaningful in discussing the interaction of the subtropical and eddy-driven jets. The presence of either a critical or turning latitude in the region between the eddy-driven and subtropical jets will prevent eddies from propagating through the subtropical jet waveguide and may lead to a persistent double jet in statistically steady state.

3. Model and experiments

We investigate the effects of a separation of forcing regions on the formation and maintenance of zonal mean jets using a barotropic doubly periodic β -plane model run at a spectral resolution with $k_{\max} = 127$. The model is modified from that of Smith et al. (2002) to include two forcing mechanisms—namely, meridional localized random stirring and Newtonian relaxation to a prescribed mean state to represent the midlatitude storm tracks and radiative cooling, respectively. The model integrates the barotropic potential vorticity equation

$$\frac{\partial q}{\partial t} + J(\psi, q) = a(y)F - \frac{1}{\tau}(\nabla^2 \psi - \nabla^2 \psi_{\text{jet}}) + D \quad (7)$$

to statistically steady state where ψ is the dynamic streamfunction, ψ_{jet} is the streamfunction of the prescribed basic state jet, $a(y)$ is the amplitude of the domain stirring field F , τ is the relaxation time scale, D is a resolution-dependent hyperdiffusion, and where

$$q = \nabla^2 \psi - k_d^2 \psi + \beta y \quad (8)$$

is the potential vorticity, β is the background potential vorticity gradient, and k_d is the deformation wavenumber.

The first two terms on the right-hand side of (7) represent the two primary forcing mechanisms for the generation of a zonal mean wind in the atmosphere. The first, $a(y)F$, represents the random stirring of the vorticity field by baroclinic eddies F confined within the meridionally narrow storm track $a(y)$. Similar localized

random stirring methods have been used in a number of additional studies (Vallis et al. 2004; Gerber and Vallis 2009; Chen et al. 2007; Barnes and Hartmann 2011). This stirring is generated by exciting a random wave field F over the entire domain at zonal wavenumbers between $6 \leq k \leq 13$ and small meridional wavenumbers of $k_y = 1, 2$. The specifics of F are described in Smith et al. (2002).

The domain forcing field F is then localized in y by a Gaussian amplitude function

$$a(y) = a_0 \exp \left[-\left(\frac{y}{\Delta_s} \right)^2 \right], \quad (9)$$

where a_0 is maximum amplitude of the stirring achieved at $y = 0$ and where Δ_s is the width of the stirring envelope.

The second forcing mechanism of (7) is a Newtonian relaxation back to a prescribed zonally symmetric streamfunction ψ_{jet} . This streamfunction produces a Gaussian jet of the form

$$u_{\text{jet}} = -\frac{\partial \psi_{\text{jet}}}{\partial y} = U_0 \exp \left[-\left(\frac{y - \Delta_y}{\Delta_r} \right)^2 \right], \quad (10)$$

where U_0 is the maximum amplitude of the relaxed jet, Δ_r is the width of the relaxed jet, and Δ_y is the center of the relaxed jet. Newtonian relaxation is meant to be qualitatively similar to a radiative relaxation of the atmosphere back to an eddy-free, zonal mean state with a characteristic subtropical jet as in Held and Suarez (1994). However, as this β -plane model has no subtropical region, this jet will be referred to as the relaxed jet rather than the subtropical jet.

In addition to the large-scale dissipation given by the Newtonian relaxation, we include hyperdiffusion D in the form of $D = \kappa \nabla^8 q$, where κ is a resolution-dependent hyperviscosity. This filter is applied to wavenumbers higher than $k_{\text{cut}} = 50$ to control the forward enstrophy cascade (Smith et al. 2002).

All of our results are presented in a nondimensional form based on the length of the domain and the value of the mean β . The dimensional model domain is square with values ranging $-\tilde{L}/2 \leq \tilde{x}, \tilde{y} \leq \tilde{L}/2$, where $\tilde{L}$ is the length of the domain. Here tildes denote dimensional values. The nondimensional domain is given by $-\pi \leq x, y \leq \pi$. We nondimensionalize using the following characteristic scales

$$k_d^2 = \frac{\tilde{f}^2 \tilde{L}^2}{g \tilde{H}}, \quad \beta = \frac{\tilde{\beta} \tilde{L}^2}{\tilde{U}}, \quad (11)$$

where k_d^2 is the ratio of the external deformation radius to the domain length scale, f_0 is the dimensional Coriolis

parameter at a latitude of $\tilde{\theta}$, $\tilde{L} = \tilde{L}/2\pi$, $\tilde{H}$ is the mean thickness of the layer, $\tilde{\beta}$ is the local dimensional gradient of $\tilde{f}$ at $\tilde{\theta}$, and $\tilde{U}$ is the characteristic scale of the wind velocity.

We have chosen the parameters of this study to agree roughly with the observed atmospheric jets. If we center the β plane at $\tilde{\theta} = 45^\circ$ and assume the length of the domain to be the circumference of Earth at 45° and a characteristic velocity of $\tilde{U} = 60 \text{ m s}^{-1}$, we obtain a nondimensional $\beta = 5.5$. Additional nondimensional parameters constant over the parameter sweep are given by

$$a_0 = 5.0, \quad \Delta_s = 0.08, \\ \tau^{-1} = 0.01, \quad \Delta_r = 0.12,$$

which correspond to dimensional values of

$$\tilde{a}_0 = 6.6 \text{ day}^{-2}, \quad \tilde{\Delta}_s = 360 \text{ km}, \\ \tilde{\tau}^{-1} = 0.0087 \text{ day}^{-1}, \quad \tilde{\Delta}_r = 540 \text{ km}.$$

It should be noted that the value of k_d^2 that we choose for the experiments with a finite deformation radius is chosen to show the impact of a changing deformation radius on the propagation of eddies within a two-jet system rather than one based on realistic observations. The value of k_d^2 that we choose corresponds to an extremely shallow characteristic depth of $\tilde{H} = 0.0014 \text{ m}$ following (11). If we were to use a more realistic characteristic depth of approximately 7 km, then $k_d^2 = 0.0031$. Such a small nondimensional deformation radius wavenumber produces results qualitatively similar to the infinite deformation radius experiments. We choose $k_d^2 = 30$ instead to obtain markedly different results from the infinite deformation radius cases in order to test the limits of our theory on $c_{\min}$.

Experiment description

We consider three sets of experiments with the specific model parameters varied in each integration listed in Table 1. As we are interested in the behavior of jets in statistically steady state, all integrations are run to 6000 nondimensional model days. A 450-model-day spinup is discarded.

There are two families of experiments described here and summarized in Table 1. The first varies Δ_y , or the separation distance between stirring and relaxation regions as given in (10), with a fixed infinite and finite deformation radius in integrations S00–S20 (“separation”) and D00–D15 (“deformation”), respectively. Numbers in the experiment name refer to half of the number of grid points the center of relaxation is moved off of the axis of stirring. These experiments are meant to determine if the interaction of eddies with

TABLE 1. Selected integrations and the accompanying parameters. Boldface values indicate primary parameter of interest for each experiment set.

Description	Integration	Δ_y	k_d^2	U_0
Infinite deformation radius (“S” for separation)	S00	0	0	0.8
	S05	0.25	0	0.8
	S15	0.74	0	0.8
	S20	1.00	0	0.8
Finite deformation radius (“D” for deformation)	D00	0	30	0.8
	D05	0.25	30	0.8
	D15	0.74	30	0.8
Relaxed jet amplitude (“R” for relaxation)	R01	0.89	0	0.1
	R05	0.89	0	0.5

the relaxed jet tends to lead to a merger or splitting of the two jets.

The primary experiment set is given by integrations S00–S20 in Table 1. In these experiments Δ_y is increased until a double jet is obtained in statistically steady state for a fluid with an infinite deformation radius. The relationship between the maximum and minimum phase speed bounds as barriers to meridional eddy propagation are considered in the context of the splitting of the jet. We find that the character of wave propagation and breaking in each of these experiment sets depends on the wavelength of the wave. As such, we analyze long waves of wavenumber $1 \leq k \leq 5$ separately from short waves of wavenumber $6 \leq k \leq 15$.

The second family of experiments is a subset of the S00–S20 runs. These integrations, R01 and R05 (“relaxation”), involve changing the amplitude of the relaxed jet U_0 at a fixed offset distance Δ_y to determine the sensitivity of the location of short wave breaking to a relatively small-amplitude relaxed jet. The naming convention of these relaxation experiments includes a number that refers to the amplitude of U_0 as can be seen in Table 1.

Both forcing mechanisms in this idealized model, stirring and relaxation, can independently create a jet. To compare the interaction of the eddy-driven and relaxed jets in the context of wave propagation, we first consider the superposition of an independently formed eddy-driven and relaxed jet. The “superimposed” control case is created by running the meridionally localized stirring and zonal mean relaxation forcing mechanisms independently until an individual eddy-driven or relaxed jet is achieved in statistically steady state. The profiles of these two time-averaged jets are then summed to create a composite mean zonal flow. This zonal flow is meant to represent a basic zonal wind profile achieved if there were no interactions between the eddies of the stirred jet and that of the relaxed jet.

The steady state eddy-driven and relaxed jets for integrations S00–S20 are shown in black in Fig. 1. Figure 1

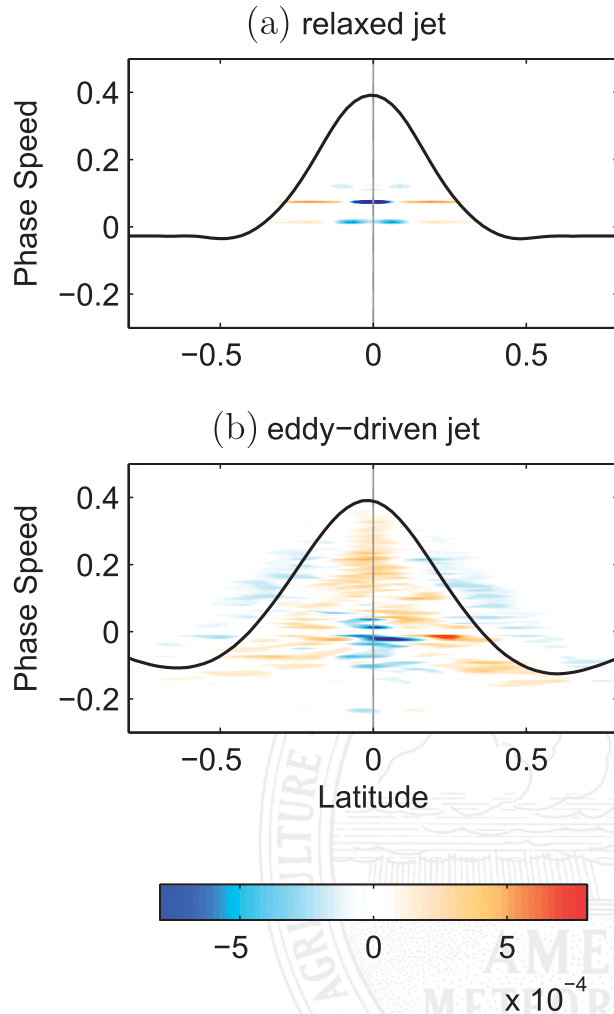


FIG. 1. Eddy momentum flux convergence cospectra (colored contours) for the (a) relaxed and (b) eddy-driven jets alone for experiments S00–S20. Time and zonal mean zonal wind associated with each jet is displayed as a thick black contour. A superposition of these control cases are used to create the superimposed wind profile (see text). All units are nondimensional.

also includes contours of eddy momentum flux convergence cospectra as a function of latitude and zonal phase speed. By design, the eddy-driven jet shows eddy activity at a range of phase speeds and over a wide range of latitudes, whereas the relaxed jet reveals relatively little eddy activity except at particular westward phase speeds. Additional details of this eddy momentum flux convergence cospectra will be discussed in section 4.

The “interacting” case, in comparison, has both forcing mechanisms active during the entire integration such that eddies generated in the region of stirring may propagate, break, or reflect off the relaxed jet, thus influencing the overall zonal mean zonal wind.

In comparing the superimposed and interacting jets of integrations S00–S20 in Fig. 2, it is evident that the interaction of the eddies with the jets leads to a tendency for the jets to merge relative to a noninteracting superposition of the jets. This tendency toward jet merger is additionally observed in experiments D00–D15 with a finite deformation radius and as the comparisons between the superimposed and interacting jet locations for runs with a finite deformation radius are similar to those with an infinite deformation radius case, results will only be shown for the infinite deformation radius experiments S00–S20.

The superimposed case suggests that, absent any interaction between the jets, we may expect to find two jets in statistically steady state at a nondimensional offset distance of approximately $\Delta_y = 0.4$ in the integrations with a finite deformation radius and $\Delta_y = 0.5$ with an infinite deformation radius. However, it is not until $\Delta_y = 0.7$ and $\Delta_y = 0.75$ that the interacting case produces two distinct jets for the finite and infinite deformation radius case runs, respectively.

Figure 2a shows the zonal mean zonal wind as a function of Δ_y for integrations S00–S20. For both the finite and infinite deformation radius cases eddies in the interacting runs tend to suppress the relaxed jet relative to the superimposed case (Fig. 2c). Additionally, Fig. 2a reveals that the transitions to a double-jet state in interacting integrations are not as abrupt as they may appear in Fig. 2d. As the center of relaxation is separated from the center of stirring, the zonal mean zonal wind broadens to a shoulder that gradually peaks into a secondary zonal wind maxima.

Nonetheless, if we define the latitude of a jet by the location of a local zonal mean zonal wind maxima, Fig. 2 shows that eddies act to merge the eddy-driven and relaxed jets. For increasing forcing offset distance, the maximum zonal wind tends to stay close to the location of the center of stirring such that it would appear the eddy-driven jet dominates. The latitude of the relaxation appears to have little influence on the latitude of the interacting eddy-driven jet in Fig. 2d, unlike that of the superimposed jet case where a more gradual movement of both the merged and individual jets can be observed.

4. Phase speed bounds

The transition from a single- to a double-jet state with increasing Δ_y is related to changes in the location of wave breaking and the formation of a region of well-mixed potential vorticity between the eddy-driven and relaxed jets. The primary mechanism by which the jets interact is by the redistribution of zonal mean angular momentum by meridionally propagating waves and eddies.

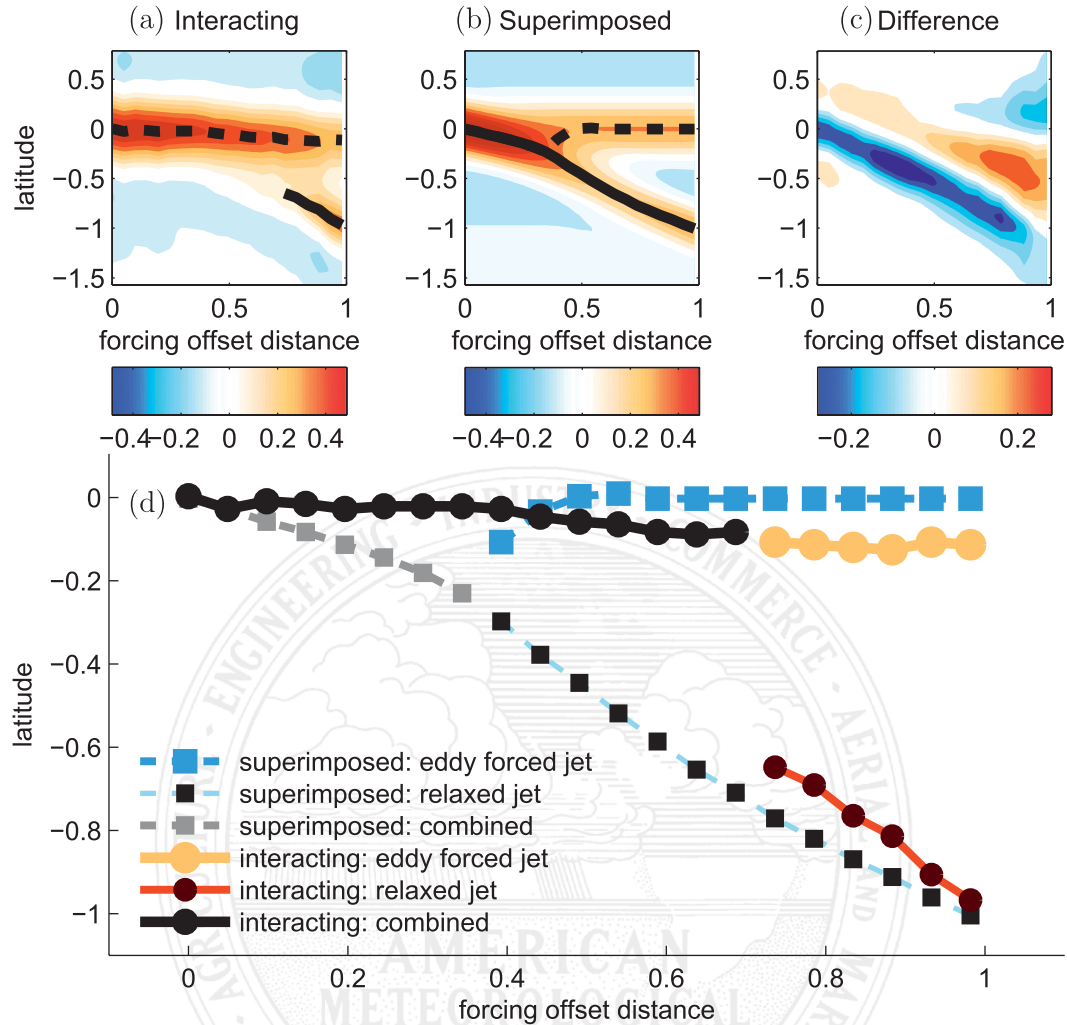


FIG. 2. Location of zonal mean zonal wind maxima as a function of offset distance between the region of maximum relaxation and maximum stirring for integrations S00–S20 with an infinite deformation radius. The region of maximum stirring is held fixed at $y = 0$. (top) Contours are of zonal mean zonal wind in statistically steady state for the (a) interacting, (b) superimposed, and (c) the difference between the interacting and superimposed cases. The locations of the eddy-driven and relaxed jets are indicated by dashed and solid black lines, respectively. (d) The comparative location of the zonal mean zonal wind maxima in statistically steady state for the interacting and superimposed cases is shown as a function of forcing offset distance. All units are nondimensional.

A summary of the eddy momentum flux convergence at long, short, and all waves for experiments S00–S20 is shown in Fig. 3 as a function of Δ_y and latitude. There is a clear compensation between the eddy momentum flux patterns of long waves and short waves that result in the total eddy momentum flux convergence of Fig. 3c.

The long-wave eddy momentum flux convergence of Fig. 3a reveals wave flux divergence at the cores of both the eddy-driven and relaxed jets with additional convergence on the cyclonic flank of the eddy-driven jet and within the interjet region. From EP flux theory, this would imply that long waves are acting to weaken the

amplitude of both jets and to accelerate the flanks, effectively broadening both jets.

On the other hand, the short-wave eddy momentum flux convergence shown in Fig. 3 differs significantly from the long-wave flux. Short waves acts to sharpen the eddy-driven jet but do little to the relaxed jet. In fact, short waves appear to avoid the relaxed jet region entirely.

Figure 3 shows that the propagation patterns of short- and long-wave eddies on the overall structure of the zonal mean zonal wind are significantly different. Long waves appear to influence both the eddy-driven and relaxed jets while short waves are confined to propagation within the eddy-driven jet alone. A more meaningful view of the

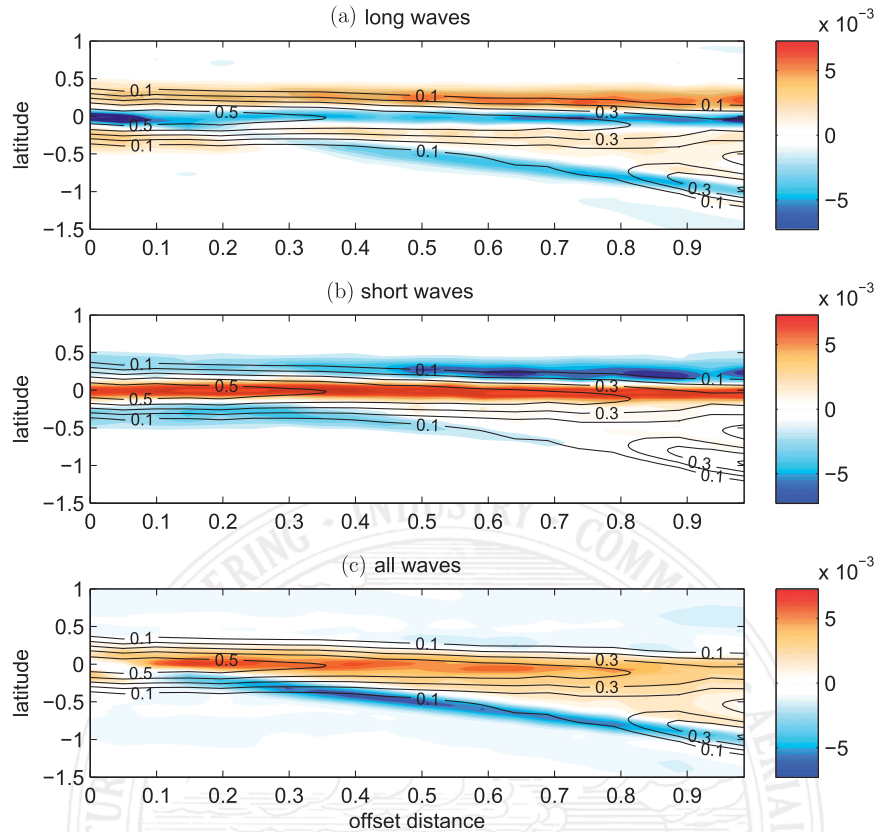


FIG. 3. Colored contours of momentum flux convergence summed over (a) long waves ($1 \leq k \leq 5$), (b) short waves ($6 \leq k \leq 15$), and (c) all waves for integrations S00–S20. Black contours are the zonal mean zonal wind with nondimensional wind magnitude labeled at intervals of 0.1 units. Latitude and offset distance are nondimensional lengths.

eddy momentum flux convergence can be obtained through the spectral analysis methods of Randel and Held (1991). We transform the eddy momentum flux convergence ($\partial/\partial y)(u'v')$ from longitude, time, and latitude to that of zonal wavenumber k , zonal phase speed c , and latitude.

[AU4] Figure 4 shows the wave-breaking pattern for the eddy momentum flux convergence summed over all wavenumbers for three integrations in S00–S20 with an infinite deformation radius and variable Δ_y . The eddy momentum flux convergence over all wavenumbers is strongly centered in the region of stirring for all Δ_y . However, the location of maximum eddy momentum flux divergence, and thus the location of wave breaking, varies as Δ_y increases.

[F4] The pattern and location of wave breaking as determined from momentum flux divergence over all wavenumbers in Fig. 4 reveal that eddies with positive phase speeds tend to break on the poleward edge of the eddy-driven jet and do not appear to propagate through the relaxed jet waveguide. The asymmetry of this breaking pattern increases with increasing Δ_y . However, the

critical lines alone do not reveal why eddies are unable to propagate into the relaxed jet region of Fig. 4.

To determine why the eddies break preferably on the cyclonic side of the eddy-driven jet, we consider the contribution to eddy momentum flux convergence by long waves ($1 \leq k \leq 5$) and short waves ($6 \leq k \leq 15$) separately. By limiting the range of wavenumbers, we can apply the lower bound on phase speed from (5) in addition to the upper bound given by $\bar{U}$.

Inclusion of the minimum bound reveals that short waves are unable to propagate into the relaxed jet region because of the presence of a turning line. In the cases where two jets are present in the time and zonal mean—namely, integrations S15 and S20 in Figs. 4c,d—the turning and critical latitude lines pinch off the interjet region and prevent eddy propagation from the region of stirring into the region of relaxation. Long waves, however, are not as confined owing to an exceptionally low minimum phase speed. It should be noted that the minimum bound for long waves in S00–S15 in Figs. 4a–c exist but are off the phase speed scale shown. In Fig. 4d the minimum phase speed bound of

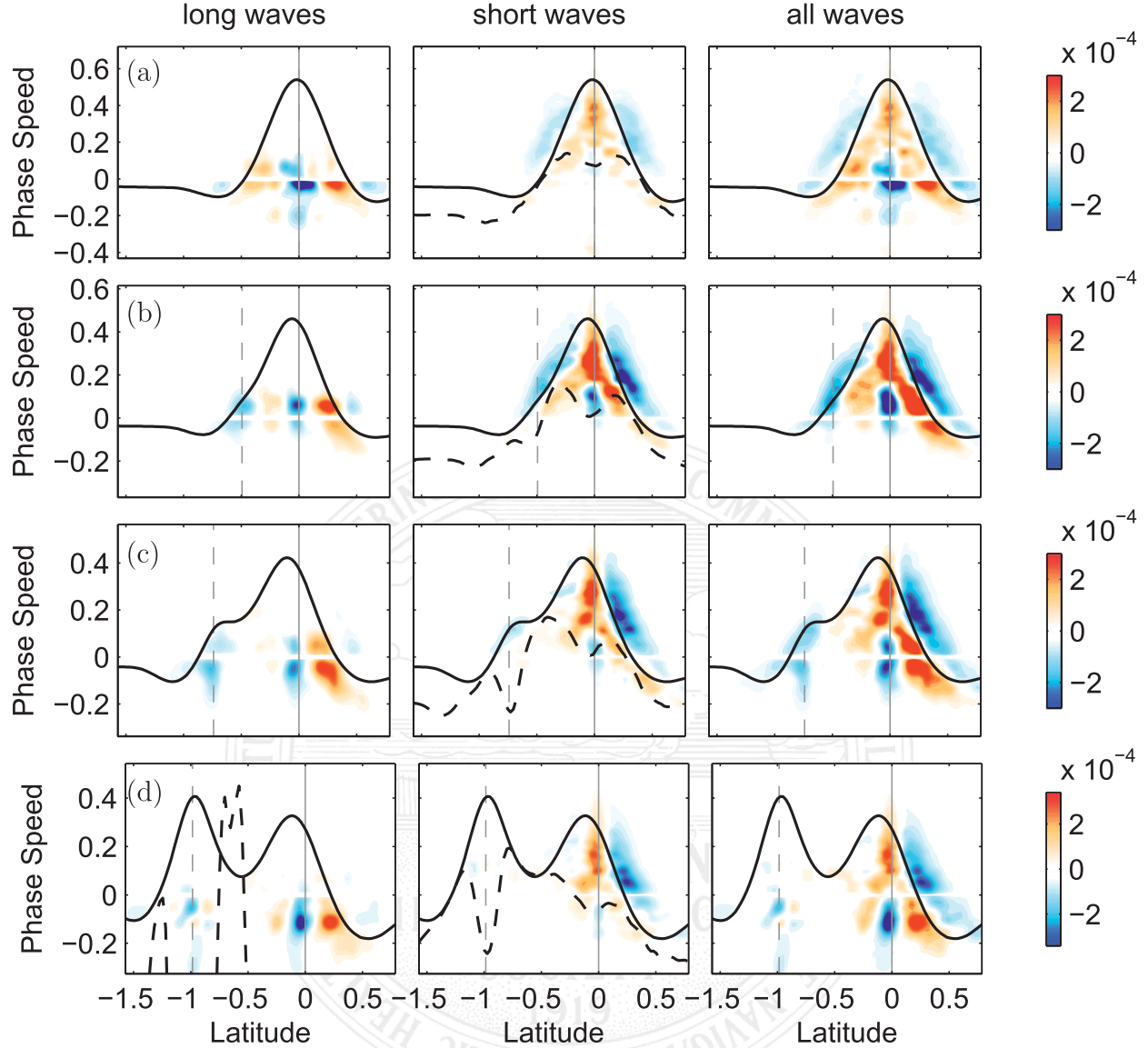


FIG. 4. Eddy momentum flux convergence cospectra for long, short, and all wavenumbers as a function of latitude and zonal phase speed for integrations increasing the separation distance with an infinite deformation radius (a) S00, (b) S10, (c) S15, and (d) S20. The maximum (solid) and minimum (dashed black) phase speed lines are given. Minimum phase speeds shown are the minimum over a range of wavenumbers considered in the long and short cases. Vertical lines indicate the central location of stirring (solid) and relaxation (dashed). Latitude, phase speed, and eddy momentum flux convergence are nondimensional.

long waves is greater than that of the upper bound along the flanks of the relaxed jet as these regions have $\bar{q} < 0$ in the time average.

The theoretical bounds on c from (5) hold to a good extent over the range of Δ_y for each wavenumber individually (not shown) in addition to over the range of wavenumbers shown in Fig. 4 for integrations S00–S20. Additionally, the minimum phase speed bound holds over the same range of Δ_y for a finite deformation radius as shown in Fig. 5.

F5

We can compare the actual minimum phase speed bound with a finite deformation radius for experiments D00–D15, as shown as a black dashed line in Fig. 5, to the hypothetical minimum phase speed bound with $k_d^2 = 0$, shown as a gray dashed line in the same figure. The finite deformation radius $k_d^2 \neq 0$ acts to increase the minimum phase speed of eddies in experiments D00–D15 given the same $\bar{U}$ and $\bar{q}_y$, particularly in the case of long waves and along the flanks of the jet. Within the jet core, however, the minimum phase speed of short waves

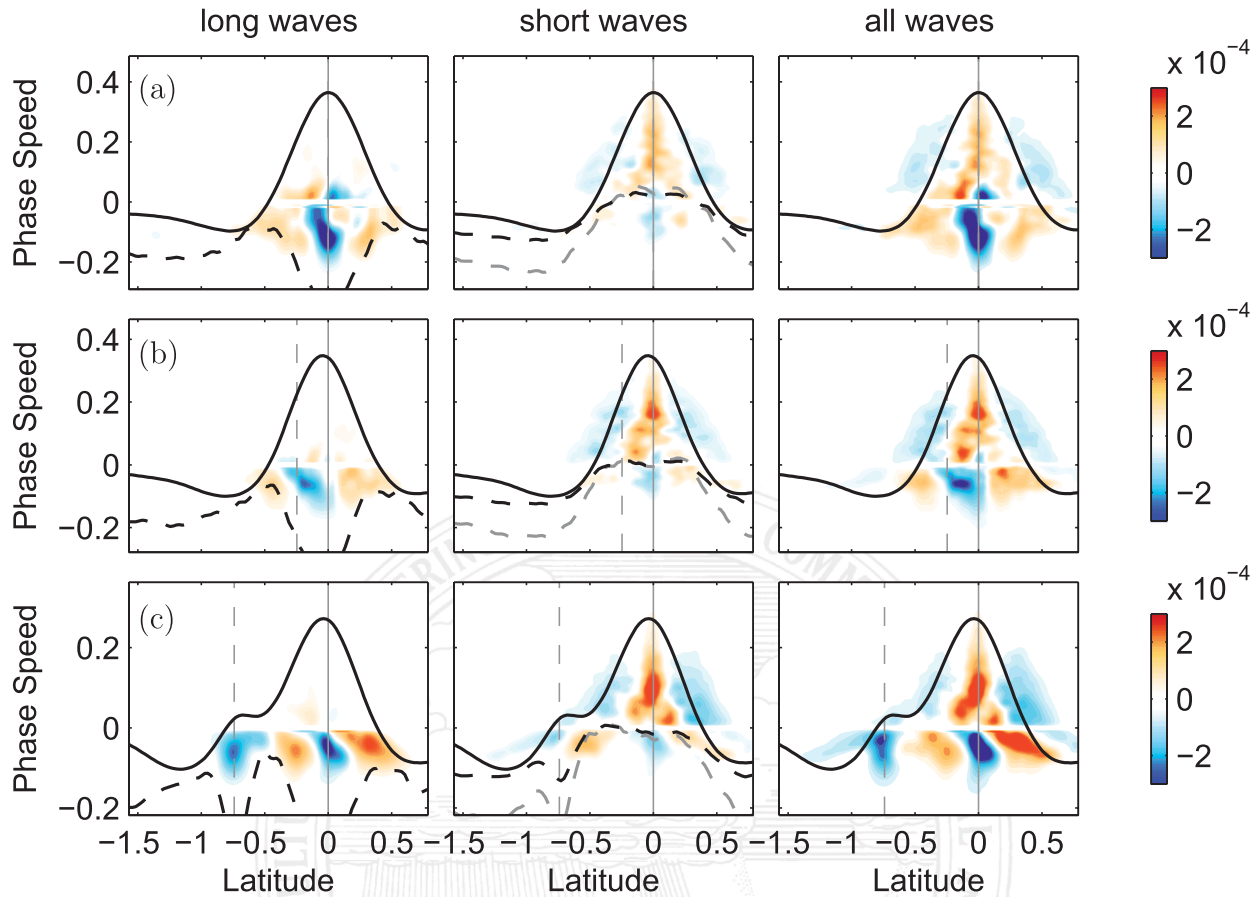


FIG. 5. Eddy momentum flux convergence cospectra for long, short, and all wavenumbers for integrations increasing the separation distance with a finite deformation radius (a) D00, (b) D05, and (c) D15. The maximum (solid) and minimum (dashed black) phase speed lines are given. Minimum phase speeds shown are the minimum over a range of wavenumbers considered in the long and short cases. In addition to the minimum phase speed for an infinite deformation radius given the same background potential vorticity profile (gray dashed). Vertical lines indicate the central location of stirring (solid) and relaxation (dashed). Latitude, phase speed, and eddy momentum flux convergence are nondimensional.

is nearly the same with and without a finite deformation radius.

The inclusion of a finite deformation radius in D00–D15 results in little change in the overall eddy momentum flux cospectra as compared to the results with an infinite deformation radius case of S00–S20. In statistically steady state, the primary difference between experiments S00–S20 in Fig. 4 and experiments D00–D15 in Fig. 5 is a reduction in the amplitude of $\bar{U}$ in D00–D15. Changes in the maximum and minimum phase speed bounds then follow changes in the equilibrium state.

The structure of turning lines of the long waves in D00–D15 (Fig. 5) and in both the short and long waves of S00–S20 (Fig. 4) show a bowl shape to the minimum phase speed within the stirring region. This increase in the range of permissible zonal phase speeds within the jet core can be attributed to the negative curvature of the zonal mean zonal wind. The large difference in $c_{\min}$ and $c_{\max}$ is then

what makes the jet core a region of enhanced eddy propagation or, in other words, a waveguide.

The region between the jets, however, has a very small background potential vorticity gradient that is indicative of a potential vorticity mixing region (Dritschel and McIntyre 2008). The smallness of the potential vorticity gradient in the interjet region implies that $c_{\min} \approx c_{\max}$ from (6) and the window of wave propagation is cut off in the interjet region, particularly for the short waves given in Fig. 4 and Fig. 5. This “pinching off” of permissible zonal phase speeds within the interjet region does not, however, remove the possibility of waves tunneling through this region. Some small amplitude wave breaking may be evident on the anticyclonic side of the relaxed jet; however, most of the contribution to eddies propagating between the eddy-driven and relaxed jet waveguides can be attributed to the long waves.

One other point may be made in considering the bounds on zonal phase speed. On the occasion that there is a change in sign of the potential vorticity gradient, particularly within the interjet region, the theoretical bounds no longer hold. In this instance $c_{\min} > c_{\max}$ owing to the strong positive meridional curvature of the zonal mean zonal wind within the interjet region. A negative background potential vorticity gradient can be observed in a number of our double-jet simulations in the time and zonal mean and suggests that barotropic instability may play a role in the dynamics of the double-jet system.

a. Long waves and barotropic instability

The eddy momentum flux cospectra for long waves at zonal wavenumbers $1 \leq k \leq 5$ in Fig. 4 for separation experiments integrations with an infinite deformation radius S00–S20 and in Fig. 5 for integrations with a finite deformation radius D00–D15 have the same characteristic pattern of eddy momentum flux divergence at the jet core and a convergence along the jet flanks. This pattern can be attributed to intermittent barotropic instability along the jet flanks. Although the time and zonal mean zonal wind is stable to barotropic instability for integrations S00–S20 and D00–D15 following Kuo (1949), the zonal mean potential vorticity gradient occasionally reverses sign in time. The time mean potential vorticity gradient for S20, in comparison, is weakly negative in the interjet region as evident in Fig. 4d wherein the minimum phase speed bound is greater than the maximum phase speed bound within the interjet region.

The potential for barotropic instability is calculated by taking the zonal mean potential vorticity as a function of time $\bar{q}_y(t)$ and counting the instances at which the potential vorticity is negative at a given latitude. The frequency that the local PV gradient satisfies the Rayleigh criterion for barotropic instability is shown as the thick black line in Fig. 6 for integrations S15 and S20. Additionally, the rescaled long-wave eddy momentum flux convergence (dashed, positive shaded) and the rescaled zonal mean zonal wind (thin gray) are shown in Fig. 6 to compare the relative location of the most frequent reversals of the background PV gradient to the regions of strong zonal wind shear and positive long-wave eddy momentum flux convergence.

From Fig. 6, the most frequent occurrence of a reversal of the zonal mean potential vorticity gradient can be found at the jet flanks, or in regions of greatest zonal mean wind shear, as would be expected. The long-wave eddy momentum flux convergence tends to be collocated with the regions of most frequent $\bar{q}_y < 0$, further suggesting that the long waves are excited by barotropic instability of the mean flow.

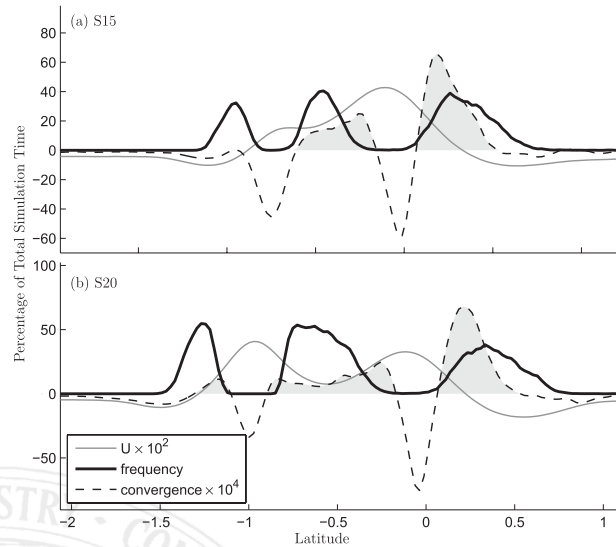


FIG. 6. Frequency of negative zonal mean potential vorticity anomalies as a function of latitude (solid) in integrations (a) S15 and (b) S20 are superimposed on long-wave contributions of time and zonal mean eddy momentum flux convergence (dashed). Positive values of eddy momentum flux convergence are shaded and amplified by a magnitude of 10^4 . For comparison the time and zonal mean zonal wind is given in light gray and amplified by a magnitude of 10^2 . Zonal wind speed, eddy momentum flux convergence, and latitude are given in nondimensional units.

Additional evidence for barotropic instability comes from the cospectra of the relaxed jet of integrations S00–S20 in the absence of an eddy-driven jet as seen in Fig. 1a. Even without random stirring, the relaxed jet produces an eddy momentum flux convergence with characteristics similar to that of the long waves of integrations S00–S20 and with a pattern similar to what would be expected from barotropic instability—namely, wave breaking at the core of the jet and a relative acceleration of the zonal mean zonal wind along the flanks of the jet. Barotropic instability acts to decelerate the jet core and accelerate the jet flanks, thus broadening the jet and reducing the magnitude of the curvature of $\bar{U}_{yy}$.

For both the finite and infinite deformation radius experiments the long waves generated by barotropic instability tend to be centered around a zonal phase speed $c = 0$ with wavelengths ranging between $k = 2$ and $k = 6$. Additional sets of experiments varying the strength of the stirring amplitude, the amplitude of the relaxed jet, and the relaxation time scale show a similar tendency toward barotropic instability at near-stationary phase speeds and long wavelengths.

These long waves, regardless of whether they originate from barotropic instability, have a marked impact on the interaction of the two-jet system. Namely, these long waves are able to interact with both the eddy-driven and relaxed jets owing to their propagation characteristics.

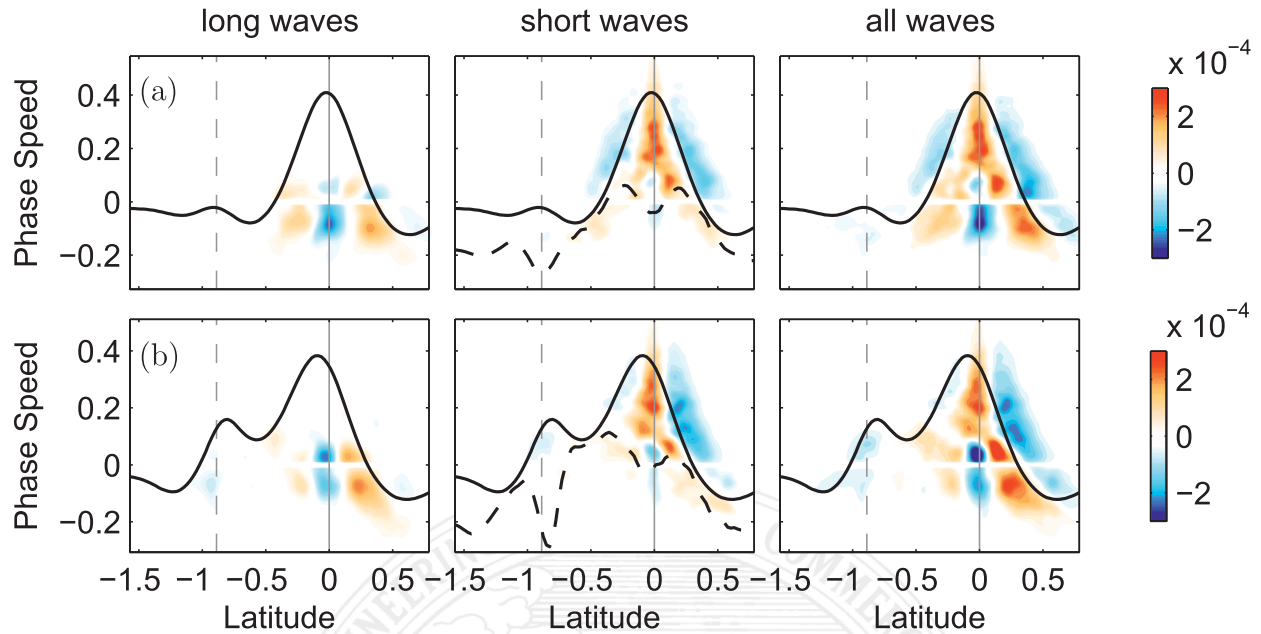


FIG. 7. Eddy momentum flux convergence cospectra for long, short, and all wavenumbers for integrations with a varying relaxed jet strength (a) R01 and (b) R05. The maximum (solid) and minimum (dashed black) phase speed lines are given. Minimum phase speeds shown are the minimum over a range of wavenumbers considered in the long and short cases. Vertical lines indicate the central location of stirring (solid) and relaxation (dashed). Latitude, phase speed, and eddy momentum flux convergence are nondimensional.

Following (5), eddies propagate between the eddy-driven and relaxed jet waveguides more readily when $c_{\min}$ is well separated by $c_{\max}$ within the interjet region. For the same $\bar{U}$, the greatest difference between maximum and minimum phase speeds occurs for small values of k . The interjet region is thus more transparent to longer wavelengths. As a result of this transparency, long waves can interact with both the eddy-driven and relaxed jets, unlike short waves, which are more confined to the eddy-driven jet.

Consider, for example, the total eddy momentum flux convergence of long waves in the separation experiments S00–S15 in Fig. 3. Long waves contribute a positive eddy momentum flux convergence in the interjet region of these integrations, particularly for integrations S15 and S20 that have a double jet in statistically steady state. Therefore, long waves tend to blend the two jets into a broad zonal wind maxima similar to that expected of barotropic instability.

b. Short waves and asymmetric wave breaking

Unlike the case with long waves, the interjet region is not transparent to short waves. The minimum phase speed bound limits the propagation of eddies from the eddy-driven jet into the relaxed jet waveguide. As a result, the eddy momentum flux divergence for short waves tends to be concentrated on the cyclonic flank of the eddy-driven jet as shown for integrations S00–S15

in Fig. 4 and for integrations D00–D15 in Fig. 5. The maximum short-wave eddy momentum flux convergence tends to be located in the core of the eddy-driven jet at the center of maximum stirring.

This pattern of eddy momentum flux convergence and divergence indicates that short waves, unlike long waves, tend to accelerate and sharpen the eddy-driven jet. This tendency and pattern of eddy momentum flux convergence is familiar as it appears frequently in eddy momentum flux diagnostics of the midlatitude upper troposphere. These dynamics are associated with the “upgradient” momentum flux of the horizontal Eliassen–Palm flux (Vallis 2006).

Because of the lower bound on the zonal phase speed, however, slow short waves propagating toward the relaxed jet tend to encounter a turning line before reaching a critical line. These slow, short waves are then reflected back into the eddy-driven jet waveguide and break preferentially along the cyclonic jet flank.

Asymmetric wave breaking due to reflection of eddies off of the relaxed jet is additionally investigated in experiments R01–R05 from Table 1. These experiments vary the amplitude of the relaxed jet while holding Δ_y and a variable U_0 fixed. The offset distance chosen for these integrations is one in which a definite double jet appears in the time and zonal mean winds of S00–S15.

Figure 7 R01–R05 shows that short waves break preferentially on the cyclonic edge of the eddy-driven

[F7]

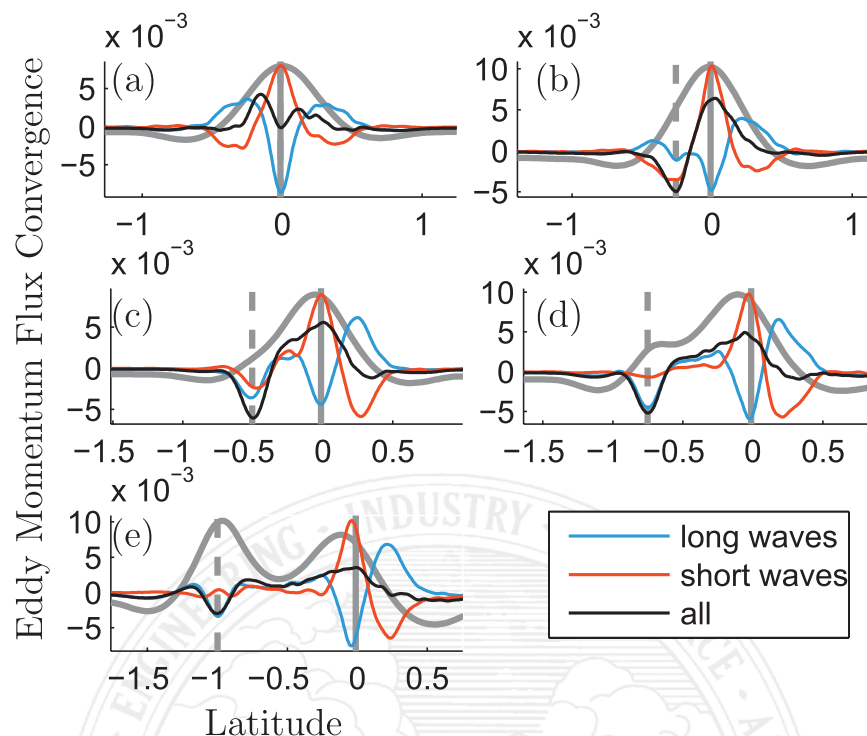


FIG. 8. Contribution to the total momentum flux convergence (black) from long waves $1 \leq k \leq 5$ (blue) and short waves $6 \leq k \leq 15$ (red) for experiments with increased separation distance and an infinite deformation radius (a) S00, (b) S05, (c) S10, (d) S15, and (e) S20. Eddy momentum flux convergence in each case is plotted against nondimensional latitude. Time and zonal average zonal wind and the location of stirring (solid) and relaxation (dashed) are in gray. Units are nondimensional.

jet even for small amplitudes of the relaxed jet. For short waves in Fig. 7b, the minimum phase speed is higher in the interjet region than on the cyclonic flank of the eddy-driven jet. Short waves traveling near to the minimum zonal phase speed will then only encounter a critical line on the cyclonic flank of the eddy-driven jet as the anticyclonic flank is blocked by the interjet turning latitude.

The eddy momentum flux convergence summed over all wavenumbers for the relatively small amplitude relaxed jet of integration R01 of Fig. 7a reveals a nearly symmetric momentum flux divergence on both the cyclonic and anticyclonic side of the eddy-driven jet, suggesting eddies propagate equally toward the cyclonic and anticyclonic flank of the jet when the relaxation is weak.

However, increasing the amplitude of the relaxed jet to that of integration R05 in Fig. 7b shows that even a modest amplitude relaxed jet shifts the center of maximum divergence to the cyclonic flank of the eddy-driven jet. The amplitude of the relaxed jet in this experiment is still smaller than that of the infinite deformation radius separation experiments S00–S15 yet the influence of the

relaxed jet is evident in the asymmetric breaking pattern of the short waves.

c. All waves

As can be seen in Fig. 8 for S00–S20, the eddy momentum flux divergence summed over all phase speeds reveals that the asymmetric breaking pattern of short waves is compensated by the eddy momentum flux convergence owing to the action of long waves. Similarly, the eddy momentum flux convergence at the center of the eddy-driven jet due to short waves is compensated by the divergence due to long waves. The resulting total eddy momentum flux convergence pattern is positive in the center of the stirring region owing to the action of short waves confined to the eddy-driven jet waveguide and negative near the core of the relaxed jet owing to the action of long waves and barotropic instability. Notably, the eddy momentum flux convergence in the interjet region is primarily due to the action of long waves.

By considering only the eddy momentum flux convergence pattern summed over all waves and all phase speeds, it would appear that waves generated in the core of the stirring region are breaking in the center of the

relaxed jet before reaching their critical line on the jet's anticyclonic flank. The usefulness of the lower bound on phase speed, and thus the wavenumber-dependent reflection of short waves off the relaxed jet, is lost when all wavelengths are considered.

5. Discussion and conclusions

We have investigated the interaction of analogs of the subtropical and eddy-driven jets in a barotropic, β -plane model. The subtropical jet was maintained against friction by relaxation of the background potential vorticity while the eddy-driven jet was generated by meridionally localized stochastic stirring (i.e., by a wavemaker) of the potential vorticity field. If the forcing regions are not too far apart, we find that the jets tend to merge relative to a superposition of the zonal mean zonal winds independently created by each forcing mechanism alone. This tendency toward the merger of the eddy-driven and subtropical jets is similar to that observed in more complex models (Son and Lee 2005; Lee 1997; Kim and Lee 2004; Yoo and Lee 2010). The idealized model also displays asymmetric wave breaking owing to the presence of the relaxed jet, thus owing to meridional asymmetries in the effective β similar to that of Orlanski (2003) in the absence of spherical geometry. We find that short waves preferentially break on the cyclonic side of the eddy-driven jet. While these short waves tend to be reflected off the subtropical jet, long waves appear to break anticyclonically at the core of the relaxed jet.

The merger of jets in statistically steady state and the asymmetric nature of short wave breaking within this model can be associated with changes in the meridional propagation characteristics of eddies due to modifications to the Rossby wave refractive index. We cast the refractive index as a function of the zonal phase speed and zonal wavenumber such that a latitude-dependent minimum phase speed can be included in wavenumber-phase speed cospectral plots of eddy momentum flux convergence similar to those of Randel and Held (1991). We then bound the phase speed of a wave or a group of waves in such a way as to describe both the merger of the eddy-driven and relaxed jets in statistically steady state and the observed asymmetric breaking pattern. As presented here, we find that eddy momentum flux convergence as a function of deformation radius, zonal wavenumber, zonal phase speed, and latitude is, to a good approximation, bounded by the maximum and minimum phase speeds of (5).

Short waves of zonal wavenumber $6 \leq k \leq 15$ have a characteristic eddy momentum flux pattern similar to that expected from meridionally propagating, baroclinically generated waves. These short waves are more likely to

encounter turning latitudes within the interjet region than long waves based on the wavenumber-dependent phase speed bounds presented in (5). In contrast, long waves of zonal wavenumber $1 \leq k \leq 5$ have a pattern of eddy momentum flux convergence similar to what would be expected from barotropic instability. From the phase speed bounds given in (5), the interjet region is more transparent to long than short waves.

Convergence of long-wave eddy momentum flux between the two forcing regions suggests that long waves tend to blend the eddy-driven and relaxed jets. These long waves act to decelerate the jet core and accelerate the jet flank, broadening both the eddy-driven and relaxed jets. Short waves eddy momentum flux acts instead to sharpen the eddy-driven jet as the minimum phase speed bound prevents short waves from entering the relaxed jet waveguide.

From the minimum phase speed bounds given in (5), short waves tend to be faster than long waves. We can then consider these results in terms of zonal phase speed instead of zonal wavelength in order to compare with more complex models. Son and Lee (2005) find that slow waves tend to blend the eddy-driven and subtropical jets while fast waves tend to separate the eddy-driven jet from the subtropical jet—a result discussed also found elsewhere in the literature (Lee 1997; Kim and Lee 2004). This agrees well with the observed eddy momentum fluxes from long (slow) and short (fast) waves in this relatively simple model.

However, the very simple model and dynamics discussed here may overestimate the importance of barotropic instability in more complex models and observations. The distinct behavioral difference between short waves of $6 \leq k \leq 15$ and long waves of $1 \leq k \leq 5$ is likely due to the choice of stirred wavenumbers at $6 \leq k \leq 13$. While short waves are directly forced through stirring, long waves can only arise through the secondary barotropic instability of this idealized model. Were we to increase the wavelength of stirred eddies, such as by forcing over the broader range of $1 \leq k \leq 13$, we would likely see that the pattern of eddy momentum flux convergence in long waves would look similar to that of short waves. As Son and Lee (2005) find that slow, long waves tend to blend the jet even in a more complex model, it is likely that the behavior of long waves is not solely driven by barotropic instability.

It is unclear if the barotropic instability mechanism generating long waves in this simple model is as relevant in more realistic settings; however, recent studies such as Kidston and Vallis (2010) suggest that barotropic instability may be an important factor in the structure of midlatitude jets. It is of future interest to determine the role of barotropic instability, if any, on the dynamics of the midlatitude tropospheric circulations.

Finally, turning latitudes played a critical role in determining the propagation characteristics of short waves in this model; however, the minimum phase speed bound that determines the location of these turning latitudes is sensitive to the structure and geometry of the mean meridional potential vorticity gradient and velocity fields. It is for future work to determine the bounds on meridionally propagating phase speeds as in (5) for stratified, spherical flows and whether turning lines have a significant effect on the splitting of jets in more complex models and observations.

Acknowledgments. This material is based upon work supported by the National Science Foundation under Grant AGS-1144302. We also thank two anonymous reviewers for their very helpful comments.

REFERENCES

- Barnes, E. A., and D. L. Hartmann, 2011: Rossby wave scales, propagation, and the variability of eddy-driven jets. *J. Atmos. Sci.*, **68**, 2893–2908.
- Benedict, J. J., S. Lee, and S. B. Feldstein, 2004: Synoptic view of the North Atlantic Oscillation. *J. Atmos. Sci.*, **61**, 121–144.
- Chen, G., I. M. Held, and W. A. Robinson, 2007: Sensitivity of the latitude of the surface westerlies to surface friction. *J. Atmos. Sci.*, **64**, 2899–2915.
- Dickinson, R. E., 1969: Theory of planetary wave–zonal flow interaction. *J. Atmos. Sci.*, **26**, 73–81.
- Dritschel, D. G., and M. E. McIntyre, 2008: Multiple jets as PV staircases: The Phillips effect and the resilience of eddy-transport barriers. *J. Atmos. Sci.*, **65**, 855–874.
- Gallego, D., P. Ribera, R. Garcia-Herrera, E. Hernandez, and L. Gimeno, 2005: A new look for the Southern Hemisphere jet stream. *Climate Dyn.*, **24**, 607–621.
- Gerber, E., and G. Vallis, 2009: On the zonal structure of the NAO and annular modes. *J. Atmos. Sci.*, **66**, 332–352.
- Harlander, U., 2002: Rossby waves in zonal barotropic flows with pseudocritical levels. *J. Atmos. Sci.*, **59**, 2665–2680.
- , H.-J. Schönfeldt, and W. Metz, 2000: Rossby waveguides in high-latitude shear flows with boundaries. *J. Geophys. Res.*, **105** (C7), 17 063–17 078.
- Hayashi, Y., 1971: A generalized method of resolving disturbances into progressive and retrogressive waves by space Fourier and time cross-spectral analyses. *J. Meteor. Soc. Japan*, **49**, 125–128.
- Held, I., and A. Hou, 1980: Nonlinear axially symmetric circulations in a nearly inviscid atmosphere. *J. Atmos. Sci.*, **37**, 515–533.
- , and M. Suarez, 1994: A proposal for the intercomparison of the dynamical cores of atmospheric general circulation models. *Bull. Amer. Meteor. Soc.*, **75**, 1825–1830.
- Hoskins, B., and T. Ambrizzi, 1993: Rossby wave propagation on a realistic longitudinally varying flow. *J. Atmos. Sci.*, **50**, 1661–1671.
- Kidston, J., and G. K. Vallis, 2010: Relationship between eddy-driven jet latitude and width. *Geophys. Res. Lett.*, **37**, L21809, doi:10.1029/2010GL044849.
- Kim, H.-k., and S. Lee, 2004: The wave–zonal mean flow interaction in the Southern Hemisphere. *J. Atmos. Sci.*, **61**, 1055–1067.
- Kuo, H.-l., 1949: Dynamic instability of two-dimensional non-divergent flow in a barotropic atmosphere. *J. Atmos. Sci.*, **6**, 105–122.
- Lee, S., 1997: Maintenance of multiple jets in a baroclinic flow. *J. Atmos. Sci.*, **54**, 1726–1738.
- , and H.-k. Kim, 2003: The dynamical relationship between subtropical and eddy-driven jets. *J. Atmos. Sci.*, **60**, 1490–1503.
- Limpasuvan, V., and D. L. Hartmann, 2000: Wave-maintained annular modes of climate variability. *J. Climate*, **13**, 4414–4429.
- Magnusdottir, G., and C. C. Walker, 2000: On the effects of the Hadley circulation and westerly equatorial flow on planetary-wave reflection. *Quart. J. Roy. Meteor. Soc.*, **126**, 1–22.
- Orlanski, I., 2003: Bifurcation in eddy life cycles: Implications for storm track variability. *J. Atmos. Sci.*, **60**, 993–1023.
- Pedlosky, J., 1986: *Geophysical Fluid Dynamics*. Springer, 636 pp.
- Randel, W. J., and I. M. Held, 1991: Phase speed spectra of transient eddy fluxes and critical layer absorption. *J. Atmos. Sci.*, **48**, 688–697.
- Schneider, E. K., 1977: Axially symmetric steady-state models of the basic state for instability and climate studies. Part II: Nonlinear calculations. *J. Atmos. Sci.*, **34**, 280–297.
- Simmons, A. J., J. M. Wallace, and G. W. Branstator, 1983: Barotropic wave propagation and instability, and atmospheric teleconnection patterns. *J. Atmos. Sci.*, **40**, 1363–1392.
- Smith, K., G. Boccaletti, C. Henning, I. Marinov, C. Tam, I. Held, and G. Vallis, 2002: Turbulent diffusion in the geostrophic inverse cascade. *J. Fluid Mech.*, **469**, 13–48.
- Son, S., and S. Lee, 2005: The response of westerly jets to thermal driving in a primitive equation model. *J. Atmos. Sci.*, **62**, 3741–3757.
- Thompson, R. O. R. Y., 1980: A prograde jet driven by Rossby waves. *J. Atmos. Sci.*, **37**, 1216–1226.
- Vallis, G., 2006: *Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-scale Circulation*. Cambridge University Press, 745 pp.
- , and E. Gerber, 2008: Local and hemispheric dynamics of the North Atlantic Oscillation, annular patterns and the zonal index. *Dyn. Atmos. Oceans*, **44**, 184–212.
- , —, P. Kushner, and B. Cash, 2004: A mechanism and simple dynamical model of the North Atlantic Oscillation and annular modes. *J. Atmos. Sci.*, **61**, 264–280.
- Yang, G., and B. Hoskins, 1996: Propagation of Rossby waves of nonzero frequency. *J. Atmos. Sci.*, **53**, 2365–2378.
- Yoo, C., and S. Lee, 2010: Persistent multiple jets and PV staircase. *J. Atmos. Sci.*, **67**, 2279–2295.