A Theory of the Interhemispheric Meridional Overturning Circulation and Associated Stratification

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ABSTRACT

A quantitative theoretical model of the meridional overturning circulation and associated deep stratification in an interhemispheric, single-basin ocean with a circumpolar channel is presented. The theory includes the effects of wind, eddies, and diapycnal mixing and predicts the deep stratification and overturning streamfunction in terms of the surface forcing and other parameters of the problem. It relies on a matching among three regions: the circumpolar channel at high southern latitudes, a region of isopycnal outcrop at high northern latitudes, and the ocean basin between.

The theory describes both the middepth and abyssal cells of a circulation representing North Atlantic Deep Water and Antarctic Bottom Water. It suggests that, although the strength of the middepth overturning cell is primarily set by the wind stress in the circumpolar channel, middepth stratification results from a balance between the wind-driven upwelling in the channel and deep-water formation at high northern latitudes. Diapycnal mixing in the ocean interior can lead to warming and upwelling of deep waters. However, for parameters most representative of the present ocean mixing seems to play a minor role for the middepth cell. In contrast, the abyssal cell is intrinsically diabatic and controlled by a balance between the deep mixing-driven upwelling and the residual of the wind-driven and eddy-induced circulations in the Southern Ocean.

The theory makes explicit predictions about how the stratification and overturning circulation vary with the wind strength, diapycnal diffusivity, and mesoscale eddy effects. The predictions compare well with numerical results from a coarse-resolution general circulation model.

1. Introduction

The meridional overturning circulation (MOC) and associated deep stratification are large-scale features of the oceanic circulation that are closely tied to each other and are of direct importance to the climate system: the MOC transports heat meridionally (Talley 2003) and regulates the exchange of CO_2 with the atmosphere (Sigman et al. 2010). The circulation and stratification of the upper ocean and thermocline have been extensively studied both theoretically and numerically (as described in books such as Pedlosky 1996). In some contrast, there is no mature body of accepted theory for the circulation and stratification of the middepth and abyssal ocean.

In the last few decades various mechanisms have been proposed for the maintenance of the deep[1] overturning circulation and associated stratification. Sufficient diapycnal mixing can of course maintain a deep stratification that is governed by an advective–diffusive balance, as was essentially posited by Munk (1966) and Munk and Wunsch (1998). This diapycnal mixing is, either explicitly or implicitly, a key feature of many models of both the deep circulation and the thermocline (Robinson and Stommel 1959; Stommel 1960; Welander 1971; Colin de Verdière 1989; Pedlosky 1992; and others). The downward diffusion of heat is balanced by its upward advection associated with the upwelling branch of the overturning

[1] We refer to "deep stratification" as any stratification below the main thermocline. This may be divided into middepth stratification—between about 1000 and 3000 m—and the still deeper abyssal stratification. We use similar nomenclature for the circulation.

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circulation. However, it seems that an unrealistically large diffusivity would be required for this process alone to maintain the observed deep stratification and rate of overturning circulation, and numerical models in closed basins produce very little deep stratification if diapycnal mixing is realistically small (Salmon 1990; Samelson and Vallis 1997), even in the presence of mesoscale eddies (Henning and Vallis 2004): the abyss simply fills up with the densest available water, leading to an unstratified abyss.

The presence of wind forcing in a southern circumpolar channel can change this picture qualitatively, as envisioned by Eady (1957) and independently proposed by Toggweiler and Samuels (1998) and Webb and Suginohara (2001). In this model, North Atlantic Deep Water will not necessarily upwell diffusively in the subtropics through the main thermocline but will cross into the Southern Hemisphere and upwell in the Southern Ocean driven by wind. Gnanadesikan (1999) proposed an elegantly simple model for this circulation. However, the model is algebraic, based on phenomenological scalings rather than the equations of motion directly, and does not include the dynamics of the abyssal cell of the overturning circulation. Our goal in this paper essentially is to develop a theory for the interhemispheric circulation that provides the following: (i) The theory should as far as possible be based on first principles (i.e., the equations of motion themselves) and predict fields that continuously vary in depth and latitude; (ii) the theory should include both the middepth cell (North Atlantic Deep Water) and an abyssal cell (Antarctic Bottom Water); (iii) it should provide quantitative predictions that can be tested against a numerical solution of the full equations and, in principle, compared with observations.

Some work along these lines has already taken place. A continuous zonally averaged theory of stratification and overturning circulation of the Southern Ocean was proposed by Marshall and Radko (2003). The theory predicts stratification and the rate of the middepth residual overturning circulation in term of the surface wind and buoyancy forcing. According to the theory, stratification in the Southern Ocean is controlled by the adiabatic advection of buoyancy by the residual circulation, which is assumed to be set in the surface mixed layer by the surface buoyancy distribution and the surface buoyancy flux. Along similar lines, Ito and Marshall (2008) applied the residual-mean framework to the abyssal cell of the Southern Ocean overturning circulation where a balance between the isopycnal advection of buoyancy by residual circulation and the cross-isopycnal diffusion by enhanced diapycnal mixing is assumed. A three-layer extension of the Gnanadesikan model to account for the abyssal cell was recently proposed by Shakespeare and Hogg (2012), and exposing the phenomenology of the interhemispheric circulation was greatly aided by a conceptual model and a series of numerical simulations by Wolfe and Cessi (2010, 2011).

The continuous models of Marshall and Radko (2003, 2006) and Ito and Marshall (2008) do not properly account for the interaction between the circumpolar channel and the rest of the ocean. To overcome this limitation, Nikurashin and Vallis (2011) presented a theory based on a matching between the nearly adiabatic dynamics in the Southern Ocean and the diabatic dynamics in the ocean basin north of it, primarily with a view to understanding the abyssal circulation. The theory shows that, although the deep stratification throughout the ocean is largely controlled by the wind and eddies in the Southern Ocean, the strength of the abyssal overturning circulation is controlled by diapycnal mixing acting across deep stratification in the ocean basin away from the Southern Ocean.

In this paper, we extend the theoretical framework of Nikurashin and Vallis (2011) to allow isopycnals to outcrop in the North Atlantic. That is, we present a theory for the deep stratification of a two-cell, interhemispheric meridional overturning circulation, crudely representing North Atlantic Deep Water (NADW) and Antarctic Bottom Water (AABW), describing both the mixing- and wind-driven components of the overturning circulation. The theory provides continuous zonally averaged solution for stratification and overturning circulation; it consistently accounts for the interaction between the circumpolar channel and the rest of the ocean, and it explicitly predicts the deep stratification and the overturning circulation in terms of the surface forcing and other problem parameters.

The paper is structured as follows: In section 2, we discuss the phenomenology of the oceanic deep stratification and meridional overturning circulation based on observations and idealized numerical simulations. In sections 3 and 4, we develop a theory for the deep stratification and overturning circulation and discuss the underlying physics using simple parametric scalings in the limits of wind- and mixing-driven overturning circulation. In section 5, the theory is compared with results from idealized numerical simulations using an ocean general circulation model (GCM). Finally, section 6 offers discussion and conclusions.

2. Phenomenology

Before describing our theory we describe some observations and results from idealized numerical simulations.

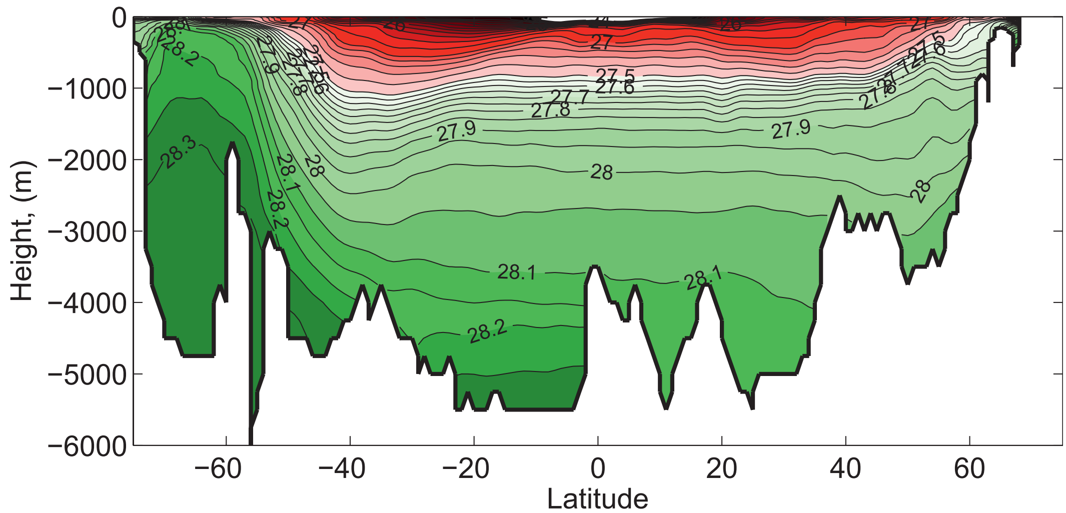

FIG. 1. Meridional section of neutral density (kg m^{-3}) in the Atlantic Ocean at 25°W from the World Ocean Circulation Experiment (WOCE). A weakly stratified water mass at middepth, roughly between the 28 and 28.1 kg m^{-3} isopycnals, corresponds to a thermostad associated with the inflow of the NADW in the Atlantic Ocean. Note that the contour intervals are 0.1 and 0.05 kg m^{-3} for isopycnals greater and lower than 27.5 kg m^{-3}, respectively.

a. Observations

The ocean is stratified throughout. The stratification is largest in the upper ocean (i.e., in the thermocline) and is small but not negligible below it (Fig. 1). In the Atlantic Ocean, isopycnals at middepth outcrop both in the North Atlantic and in the Southern Ocean, whereas isopycnals in the abyssal ocean outcrop only in the Southern Ocean. Isopycnals are nearly flat in most of the ocean basin away from the regions of isopycnal outcrop and below the region of the main thermocline where circulation is strongly affected by the low-latitude and midlatitude winds. Observations show a thermostad (a weakly stratified water mass) at middepth associated with the inflow and southward propagation of the NADW in the Atlantic Ocean.

The global zonally averaged MOC, estimated using inverse techniques incorporating various observations (Lumpkin and Speer 2007), is shown in Fig. 2. The leading-order structure of the MOC consists of wind-driven cells in the upper few hundred meters of the ocean (associated with the main gyres) and two overturning cell in the deep ocean: a middepth cell, with sinking in the North Atlantic and upwelling across isopycnals at low and middle latitudes and along isopycnals in the Southern Ocean, and an abyssal cell, with sinking around Antarctic continent and the cross-isopycnal upwelling in the abyssal ocean. The middepth and abyssal cells of the overturning circulation correspond to the formation and propagation of the NADW and AABW, respectively. Although the Atlantic Ocean has both the middepth and

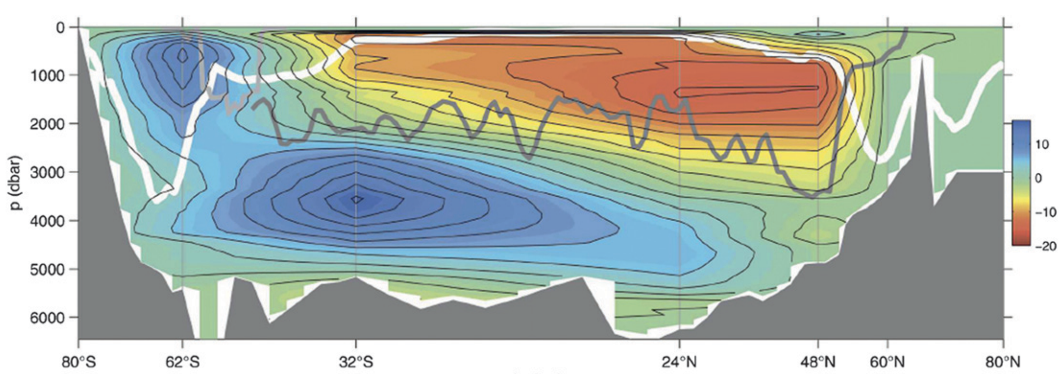

FIG. 2. Global zonally averaged MOC (Sv) from Lumpkin and Speer (2007): blue, positive (red, negative) streamlines correspond to clockwise (counterclockwise) circulation in the meridional plane.

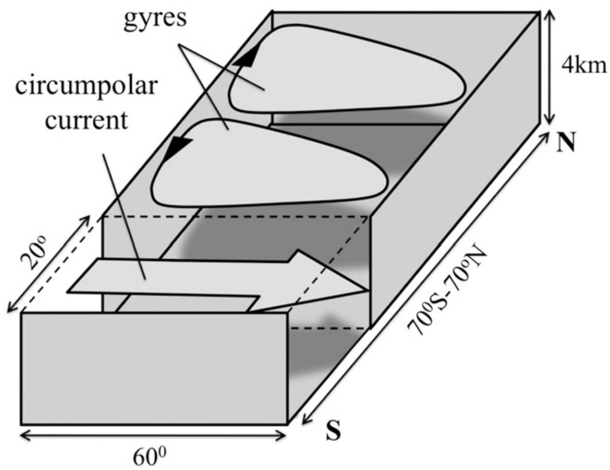

FIG. 3. Schematic of the domain used in idealized simulation. The domain consists of a rectangular basin that connects to a circumpolar full-depth channel at its southern boundary.

abyssal cells, the Indian and Pacific Oceans are characterized by only one cell.

b. Idealized simulations: Setup

We now describe some idealized simulations that we will later use to test our theory, using the Modular Ocean Model (MOM) version 4.0c (Griffies 2004). The model is configured in a single-basin flat-bottomed domain (Fig. 3) with a horizontal resolution of 2° × 2° and 20 vertical levels of thickness varying from 20 m at the top to 380 m at the bottom. The domain extends from 70°S to 70°N across 60° of longitude with a uniform depth of 4 km. There is a full-depth zonally periodic channel between 70° and 50°S. The effect of eddies is parameterized with a Gent–McWilliams-like scheme in the form of the boundary value problem (Ferrari et al. 2010). We use a uniform diapycnal diffusivity, with a value of $\kappa_\nu = 2 \times 10^{-5}\ \mathrm{m^2\ s^{-1}}$ in the control case. Linear equation of state is used.

The surface buoyancy flux is implemented via a restoring boundary condition on temperature with a restoring time scale of 10 days, and salinity is set to a uniform constant. We use a zonally uniform SST and wind stress (Fig. 4) constructed analytically based on the annual global zonal-mean observations from the National Oceanographic Data Center (NODC) *World Ocean Atlas 2001* (Conkright et al. 2002) and the last 20 yr of the National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) reanalysis (Kalnay et al. 1996) for temperature and wind stress, respectively. The model is initialized from a state of rest and is spun up over 5000 yr until it reaches a steady state. Various perturbation experiments are spun off from this

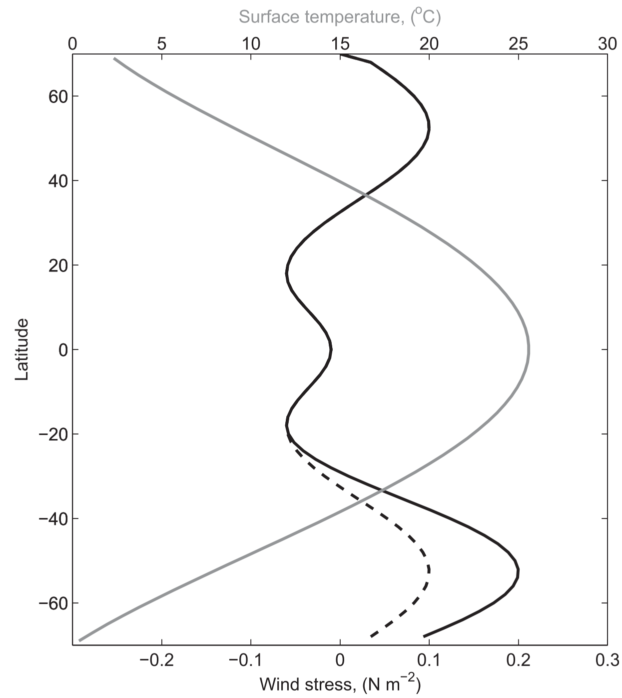

FIG. 4. The idealized mean zonally uniform wind stress (black; N m^{-2}) and surface temperature restoring field (°C; gray). A perturbed wind over the Southern Ocean is shown as a dashed black line.

control case, as described in more detail later on. Similar numerical model setups have been used to study the large-scale stratification and overturning circulation previously (e.g., Vallis 2000; Henning and Vallis 2005; Ito and Marshall 2008; Wolfe and Cessi 2010, 2011).

c. Idealized simulations: Results

Although the simulation setup is very idealized it does captures the leading-order structure of the deep stratification and overturning circulation (Fig. 5). Our focus is on an Atlantic-like ocean basin, but by changing the buoyancy boundary conditions in the Northern Hemisphere a Pacific-like simulation (with no distinct middepth cell) can be easily achieved. Stratification is concentrated in the upper ocean and extends to the middepth and abyssal ocean. The middepth isopycnals outcrop at high latitudes in the Northern Hemisphere and in the circumpolar channel, whereas the abyssal isopycnals outcrop only in the channel. Consistent with observations, the deep overturning circulation consists of two cells: a middepth cell, with sinking at the northern boundary and upwelling both at low and middle latitudes and in the circumpolar channel, and an abyssal cell, with sinking at the southern boundary and upwelling in the abyssal ocean and the circumpolar channel.

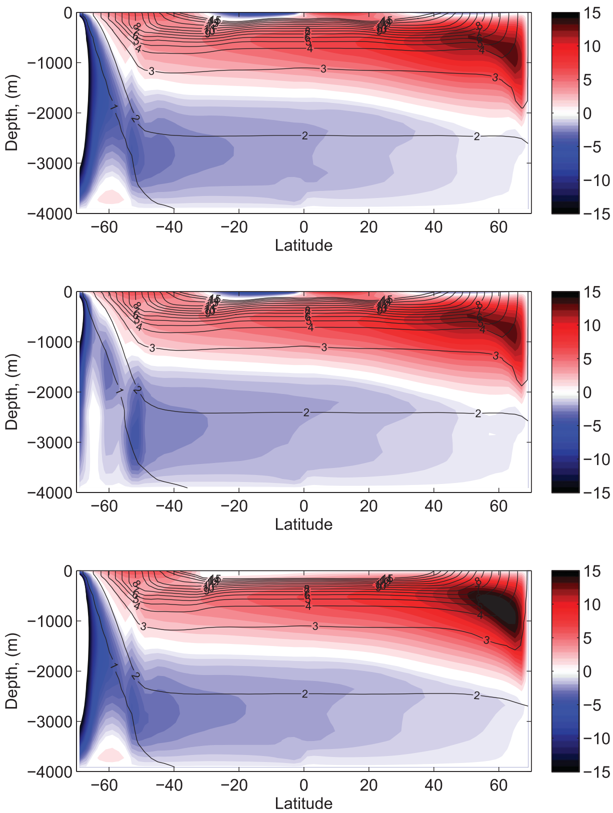

FIG. 5. Zonally averaged temperature (°C; black lines) and residual overturning streamfunction (Sv; blue/red) diagnosed from (top) an idealized GCM simulation from the control experiment, (middle) an experiment with a 1.5-km-tall ridge in the circumpolar channel, and (bottom) an experiment with zero wind stress north of 30°S.

Based on observations and numerical simulations, we distinguish three regions of the ocean that are important for the deep stratification and overturning circulation: the circumpolar channel in the Southern Hemisphere, an isopycnal outcrop region in the Northern Hemisphere, and the ocean basin between them.

In the circumpolar channel, where meridional boundaries are absent, the wind-driven circulation extends through entire water column and acts to steepen isopycnals. Mesoscale eddies, generated through baroclinic instability of the zonal flow in the ocean and parameterized in our idealized simulations, cause the isopycnals to slump and drive an eddy-induced circulation that, when added to the Eulerian wind-driven circulation, forms a residual circulation. If the flow in the channel is nearly adiabatic then the residual circulation will be along isopycnals but not necessarily zero.

In the ocean basin, away from the wind-driven surface intensified circulation, deep isopycnals are nearly flat. In a steady state, meridional flows associated with the overturning circulation are confined to the western boundary where changes in their planetary vorticity can be balanced by friction (Fig. 6). If there is sufficiently large diapycnal mixing in the ocean basin, then the downward diffusion of heat is balanced by its upward advection as in the classic advective–diffusive balance (e.g., Munk 1966). The stretching of the water column by the cross-isopycnal upwelling sets the fluid in motion and produces patterns of the deep horizontal circulation as described by Stommel (1960).

In the isopycnal outcrop region in the Northern Hemisphere (Fig. 6), isopycnals from the middepth ocean outcrop to match to the buoyancy distribution at the surface, as determined by the surface buoyancy flux. The depth of these middepth isopycnals in the ocean interior is determined by wind and eddies in the channel and to some degree by diapycnal mixing in the ocean basin (in a way we describe in detail in later sections). Thermal wind balance, associated with the meridional temperature gradient at high latitudes, leads to an eastward flow in the upper ocean and the return westward flow in the deeper ocean (Fig. 6). These zonal flows in the Northern Hemisphere are of course blocked by meridional boundaries. At the eastern boundary, the shallow eastward flow subducts and returns as a deeper westward flow. At the western boundary, the zonal flows connect to the meridional flows in the deep western boundary current. Consistent with this description, simulations show that the deepest mixed layer is in the northeast corner of the domain (Fig. 7).

The fact that the deep circulation and stratification are controlled primarily by the Southern Ocean wind and not influenced significantly by the wind stress in the enclosed part of the domain may be inferred in part from the fact that, at least in the low diffusivity limit, the main thermocline is an effective buffer between the deep circulation and the surface (Samelson and Vallis 1997). Furthermore, the main influence of the boundary conditions at high northern latitudes (e.g., in the subpolar gyre) is through the buoyancy conditions, not the stress. To demonstrate these features explicitly, we carried out an experiment in which the wind stress was set to zero north of 30°S (Fig. 5). The result shows that, although the wind-driven overturning cells in the upper ocean disappear, the deep-ocean overturning circulation and stratification remain largely unchanged.

3. Theory

Guided by observations and numerical simulations we present a simple conceptual theory of the deep stratification and overturning circulation. Our theory is for the zonally averaged stratification and circulation of the ocean below the main thermocline.

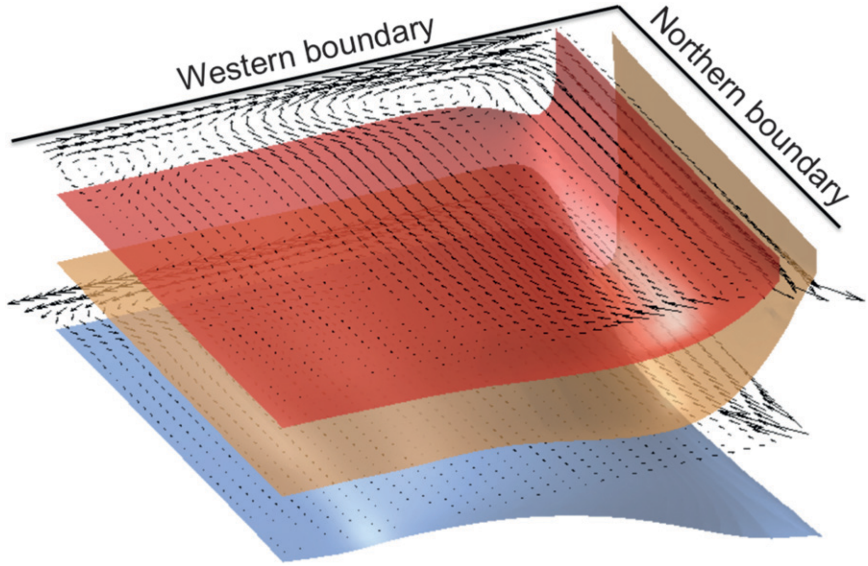

FIG. 6. Three isopycnal surfaces (color) and shallow and deep horizontal flows (black arrows) in the Northern Hemisphere from idealized GCM simulation.

Formulation

Let us consider an ocean consisting of a closed basin, which extends across the equator, connected to a circumpolar channel, as illustrated in Fig. 3. The circulation is, we assume, primarily constrained by the dynamics in three regions of the domain: the circumpolar channel in the Southern Hemisphere, the isopycnal outcrop region in the Northern Hemisphere, and the interior of the ocean basin between them (Fig. 8). The dynamics in other parts of the domain, such as the western boundary currents and convective sites at the southern and northern boundaries, are assumed to be passive and required to close the circulation. We formulate the dynamics in each of the three regions and then match the circulation and stratification between adjacent regions without boundary layers. Subscripts on the variables of 1, 2, and 3 correspond to the circumpolar channel, ocean basin, and isopycnal outcrop regions, respectively.

1) DYNAMICS IN THE CIRCUMPOLAR CHANNEL

In the circumpolar channel, where flow is not blocked by topography, the steady-state buoyancy distribution $b_1(y, z)$ is governed by the zonally averaged advection–diffusion equation

$$J(\psi_1, b_1) = \partial_z(\kappa_v \partial_z b_1), \tag{3.1}$$

where the term on the left-hand side is the Jacobian operator representing the advection of buoyancy by the residual circulation $\psi_1(y, z)$ and the term on the right-hand side is the vertical diffusion with a mixing coefficient κ_ν. If $\kappa_\nu = 0$, the residual circulation is not necessarily zero but the advection is along isopycnals and so adiabatic. Vertical diffusion is retained in the Eq. (3.1) in order to apply the surface flux boundary condition as described below. In practice, if mixing is uniform throughout the domain, then its impact on the overturning circulation in the circumpolar channel is negligible compared to the one in the ocean basin as discussed in detail in Nikurashin and Vallis (2011). Except in the case of unrealistically large diffusivities, the residual circulation in interior of the circumpolar channel is largely along isopycnals.

The residual circulation is a superposition of the Eulerian-mean circulation $\overline{\psi}$ and the eddy-induced circulation ψ^*,

$$\psi_1 = \overline{\psi} + \psi^*. \tag{3.2}$$

The Eulerian-mean circulation is given by the surface Ekman transport

$$\overline{\psi} = -\frac{\tau}{\rho_0 f}, \tag{3.3}$$

where $\tau(y)$ is the wind stress, $f(y)$ is the Coriolis frequency, and ρ_0 is a reference density, and the eddy-induced circulation is parameterized as

$$\psi^* = -K_e \frac{\partial_y b_1}{\partial_z b_1}, \tag{3.4}$$

where K_e is an eddy diffusivity. Such an eddy parameterization is chosen because it is rational, simple, and commonly used (e.g., Gent and McWilliams 1990). The

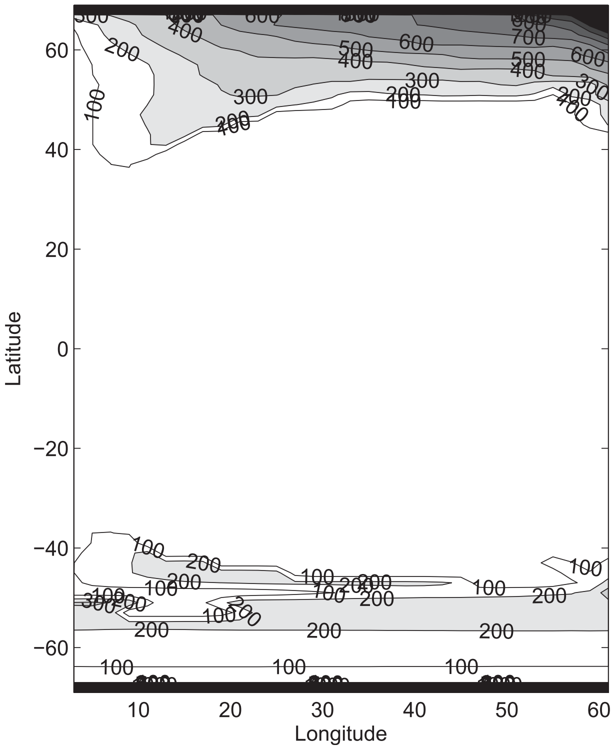

FIG. 7. Mixed layer depth (m) diagnosed from idealized GCM simulation.

theory presented here can in principle be extended to other forms of eddy parameterization.

Equations (3.1)–(3.4) describe the distribution of buoyancy b_1 and the residual overturning circulation ψ_1 within the channel. For the typical circulation in the Southern Ocean driven by the westerly winds, the surface buoyancy distribution across the channel is continuously mapped through interior of the channel to its northern edge. To complete the problem, boundary conditions at the surface and at the northern edge of the channel need to be specified. At the surface, at $z = 0$, we use the restoring flux boundary condition,

$$\kappa_v \partial_z b_1|_{z=0} = \lambda(b_s - b_1), \tag{3.5}$$

where $b_s(y)$ is the prescribed surface restoring buoyancy and λ is the restoring rate. At the northern edge of the channel, at $y = 0$, the buoyancy distribution in the channel is required to match to that in the ocean basin north of the channel: that is,

$$b_1|_{y=0} = b_2, \tag{3.6}$$

where $b_2(z)$ is the vertical buoyancy distribution in the interior of the ocean basin. Trivial boundary conditions of no flux and no normal flow are imposed at the bottom, $z = -H$, and the southern boundary of the channel, $y = -l_s$, where H is the depth of the domain and l_s is the width of the circumpolar channel.

Equations (3.1)–(3.4) and the boundary condition (3.5) and (3.6) fully constrain the buoyancy distribution and hence the residual overturning circulation in the circumpolar channel. If stratification in the ocean basin were prescribed, then the theory would become similar to a model of the upper branch of the meridional

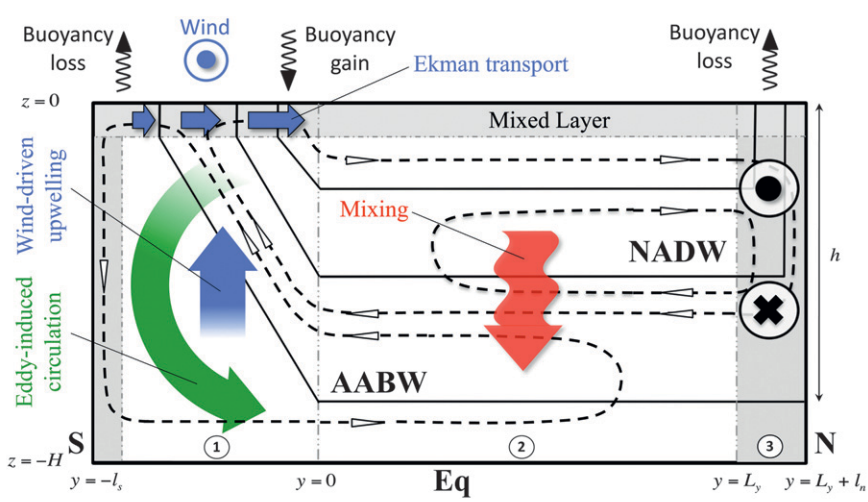

FIG. 8. Schematic of the MOC. Thin solid black lines are the isopycnals, thicker dashed black lines with arrows are the overturning streamlines of the residual circulation, dashed vertical lines are the boundaries between adjacent regions, shaded gray areas are the convective regions at high latitudes and the surface mixed layer, and the red arrow represents downward diffusive flux due to mixing uniform throughout the ocean. Labels 1, 2, and 3 (in circles) correspond to the circumpolar channel, ocean basin, and isopycnal outcrop regions considered in the theory.

circulation of Marshall and Radko (2006). However, stratification in the ocean basin north of the channel is not independent of the channel dynamics but rather influenced—or perhaps even set—by the Southern Ocean processes as suggested by various numerical simulations (Vallis 2000; Henning and Vallis 2005; Wolfe and Cessi 2010). In the section below, we describe the dynamics that governs stratification in the ocean basin and so that determines $b_2(z)$.

2) Dynamics in the ocean basin

In the interior of the ocean basin (region 2), isopycnals are assumed to be flat: that is, $b_2 = b_2(z)$. The vertical buoyancy distribution $b_2(z)$ is assumed to be governed by the vertical advection–diffusion balance,

$$w_2 \partial_z b_2 = \partial_z(\kappa_v \partial_z b_2), \tag{3.7}$$

where $w_2(z)$ is the meridional-mean vertical velocity given by

$$w_2 = \frac{\psi_3|_{y=L_y} - \psi_1|_{y=0}}{L_y}, \tag{3.8}$$

where $\psi_1|_{y=0}(z)$ and $\psi_3|_{y=L_y}(z)$ are the overturning streamfunctions at the southern and northern ends of the basin, respectively, and L_y is the meridional extent of the ocean basin. If $\psi_1 \neq \psi_3$, then the inflow/outflow of water into the ocean basin from the north is not equal, at the same level, to the outflow/inflow of water into the circumpolar channel in the south. This, in turn, implies a convergence of the horizontal transport and hence a vertical velocity that advects buoyancy in the vertical and must be, in a steady state, balanced by the vertical diffusion.

In a limit of a purely adiabatic flow, $\kappa_v = 0$, the vertical velocity in the ocean interior is zero and inflow/outflow of water in the north must be equal to outflow/inflow of water in the south, (i.e., $\psi_1 = \psi_3$). In that case, water masses entering the ocean basin cross it in the meridional direction along the western boundary, where variation in their planetary vorticity with latitude is balanced by friction, without changes in either their properties or their transports.

3) Dynamics in the isopycnal outcrop region

In the isopycnal outcrop region at high latitudes in the Northern Hemisphere (region 3), the vertical buoyancy distribution in the ocean basin $b_2(z)$ must map onto to the buoyancy distribution at the surface $b_s(y)$. To properly describe the shape of isopycnals one would need to invoke a fully three-dimensional dynamics in this region as illustrated in Fig. 6. Rather, in order to focus on the leading-order effect of outcropping isopycnals (i.e., they create a pressure gradient and drive zonal flows), we assume that the surface buoyancy distribution $b_s(y)$ matches to the buoyancy in the ocean interior $b_2(z)$ in a convective sense: that is, surface waters convect downward to the level of neutral buoyancy. Matching $b_2(z)$ and $b_s(y)$ convectively, we obtain the buoyancy distribution $b_3(y, z)$ in the isopycnal outcrop region.

Outcropping isopycnals in the Northern Hemisphere plainly result in a meridional buoyancy gradient. The gradient, as well as the corresponding meridional pressure gradient, is determined by a combination of the surface buoyancy flux forcing isopycnals to outcrop at high latitudes and the wind and interior mixing that force the isopycnals down into the ocean interior at low and middle latitudes. Thus, the zonal flow associated with the meridional buoyancy gradient is driven locally by the Northern Hemisphere buoyancy flux and remotely by the Southern Hemisphere winds and interior diapycnal mixing. For a given buoyancy distribution $b_3(y, z)$, the zonal flows can be computed using the thermal wind relation,

$$u_3(y,z) = -\frac{1}{f}\int_{-h}^{z} \partial_y b_3 \, dz' + C, \tag{3.9}$$

where h is the depth of the middepth cell of the overturning circulation determined by the depth of the first isopycnal that outcrops only in the circumpolar channel and $C(y)$ is a constant of integration computed from a requirement that, for a thermally driven flow considered here, the total vertically integrated transport is zero at every latitude: that is,

$$\int_{-h}^{0} u_3 \, dz = 0. \tag{3.10}$$

The volume transport of water associated with the eastward flow in the upper ocean and westward return flow in the deep ocean is obtained from

$$\psi_3(z) = \int_{-h}^{z} \int_{L_y}^{L_y + l_n} u_3 \, dy \, dz', \tag{3.11}$$

where l_n is the meridional extent of the isopycnal outcrop region. Finally, the eastward flow is assumed to subduct and connect to the westward return flow at the eastern boundary and both the eastward and westward zonal flows are assumed to connect to meridional flows at the western boundary (Fig. 6). Connection of zonal flows to the western boundary currents is consistent with frictional boundary layer dynamics, where changes in the meridional flow planetary vorticity is balance by friction at the western boundary, as discussed in detail in Pedlosky (1996).

4. Scalings and numerical results

The equations presented in the previous section provide a full set of continuous equations that describe the zonally averaged stratification and overturning circulation throughout the deep ocean and, taken together, constitute the full expression of our theory. The equations can be solved numerically as we describe in the next section, but first we would like to discuss the underlying physics using simple scalings that result from the theoretical equations. In particular, we present scalings for the middepth cell of the overturning circulation in two limits: a limit of adiabatic overturning circulation, when diapycnal mixing in the ocean basin is small and the circulation is primarily driven by winds over the Southern Ocean, as was proposed by Toggweiler and Samuels (1998), and a limit of diabatic overturning circulation, when the overturning circulation is primarily driven by diapycnal mixing in the ocean basin, as suggested by Robinson and Stommel (1959) and, more explicitly, Munk (1966). Scalings for the abyssal cell of the overturning circulation were previously presented and discussed in Nikurashin and Vallis (2011).

a. Scalings

Replacing terms in Eqs. (3.2)–(3.11) with their characteristic scales, we obtain the following parametric scalings:

$$\Psi_1 = \overline{\Psi} + \Psi^* = \left(\frac{\tau_0}{\rho_0 f_1} - K_e \frac{h}{l_s}\right) L_x, \qquad (4.1)$$

$$\Psi_2 = \Psi_3 - \Psi_1 = \frac{\kappa_v}{h} L_x L_y, \quad \text{and} \qquad (4.2)$$

$$\Psi_3 = \frac{\Delta b h^2}{f_3}, \qquad (4.3)$$

where Ψ_1, $\overline{\Psi}$, and Ψ^* are scales for the residual, wind-driven, and eddy-induced transports in the Southern Ocean, respectively; Ψ_2 is a scale for the mixing-driven upwelling in the ocean basin; Ψ_3 is a scale for the deep-water formation in the Northern Hemisphere; τ_0 is the characteristic wind stress in the Southern Ocean; f_1 and f_3 are the characteristic Coriolis frequencies at high southern and northern latitudes, respectively (both taken as positive); Δb is the buoyancy range for isopycnals which are shared between the circumpolar channel and the isopycnal outcrop region in the Northern Hemisphere; h is depth scale for the middepth stratification; L_x and L_y are the zonal and meridional extents of the ocean basin, respectively; and l_s is the meridional extent of the circumpolar channel. The unknowns in these equations are Ψ_1, Ψ_2, Ψ_3, and h, and there are four equations [(4.2) contains two equations] so the system is closed. We can combine the equations into a single equation for h, namely,

$$\frac{\Delta b h^2}{f_3} - \left(\frac{\tau_0}{\rho_0 f_1} - K_e \frac{h}{l_s}\right) L_x = \frac{\kappa_v}{h} L_x L_y. \qquad (4.4)$$

Our theory in this reduced form thus contains the algebraic model of the main pycnocline proposed by Gnanadesikan (1999), although our interpretation of how the Northern Hemisphere buoyancy gradient leads to an overturning circulation [i.e., produces a Ψ_3 in (4.3)] differs. We also note that typical parameter values for the middepth ocean are $\kappa_v = 10^{-5}$ m^2 s^{-1}, $K_e = 10^3$ m^2 s^{-1}, $\tau_0 = 0.2$ N m^{-2}, $f_1 = f_3 = 10^{-4}$ s^{-1}, $\rho_0 = 10^3$ kg m^{-3}, $L_x = 5000$ km, $L_y = 10\,000$ km, $l_s = 1000$ km, and $\Delta b = 10^{-2}$ m s^{-2}.

1) WEAK DIFFUSIVITY: WIND-DRIVEN LIMIT

It is instructive to consider certain limits of (4.1)–(4.4). Let us first consider a limit when the mixing-driven upwelling in the ocean basin is small compared to the wind-driven upwelling in the Southern Hemisphere: that is, $\Psi_2 \ll \Psi_1$. This limit will arise when the diapycnal diffusivity is sufficiently small, and this will be quantified later. In this limit, there is a balance between the deep-water formation at high latitudes in the Northern Hemisphere Ψ_3 and along-isopycnal wind-driven upwelling in the Southern Ocean Ψ_1. In the Southern Ocean, the streamfunction is determined by the wind driving and the compensating eddy effect. However, the wind driving cannot be unimportant, because then there would be no pulling of the water from Northern Hemisphere (no "pump" in the terminology of Samelson 2004) and the eddies themselves would have nothing to work against. Thus, let us initially assume that the cancellation effect of the eddy-induced circulation is small (i.e., $\Psi^* \ll \overline{\Psi}$); we then obtain that the rate of the adiabatic interhemispheric overturning circulation is essentially fixed by the Ekman transport in the Southern Ocean,

$$\Psi_1 = \Psi_3 = \overline{\Psi} = \frac{\tau_0}{\rho_0 f_1} L_x. \qquad (4.5)$$

Putting in the parameters listed above, we find $\Psi_3 \approx$ 10 Sv (1 Sv $\equiv 10^6$ m^3 s^{-1}). The corresponding depth scale of stratification is determined by an interhemispheric balance between the deep-water formation in the north and the wind-driven upwelling in the south, and using (4.3) and (4.5) we obtain

$$\frac{\Delta b h^2}{f_3} = \frac{\tau_0}{\rho_0 f_1} L_x, \tag{4.6}$$

which results in a depth scale h for the stratification,

$$h = \left(\frac{\tau_0 f_3 L_x}{\rho_0 f_1 \Delta b}\right)^{1/2} \tag{4.7}$$

Putting in the numbers, we find $h \approx 320$ m. Note that this should not be interpreted as the depth of the main thermocline in mid latitudes, because the direct effects of the wind create the gyres and push down isopycnals in middle latitudes, an effect we are not including here.

We may consider the more general case in which the compensating eddy effects are the same order as the wind effects in producing Ψ_1. The scaling is less satisfactory, because there are additive terms in the expression for Ψ_1. Proceeding nevertheless, using (4.1) and (4.3) with $\Psi_2 = 0$, we find

$$\left(\frac{\tau_0}{\rho_0 f_1} - K_e \frac{h}{l_s}\right) L_x = \frac{\Delta b h^2}{f_3}, \tag{4.8}$$

and solving the quadratic gives

$$h = \left(\frac{\tau_0 f_3 L_x}{\rho_0 f_1 \Delta b}\right)^{1/2} (-\alpha + \sqrt{1 + \alpha^2}), \tag{4.9}$$

where α is the nondimensional number given by the ratio of the wind to eddy effects,

$$\alpha = \frac{1}{2} \frac{K_e}{l_s} \left(\frac{\rho_0 L_x f_1 f_3}{\tau_0 \Delta b}\right)^{1/2} = \frac{1}{2} \frac{\Psi^*}{\Psi}. \tag{4.10}$$

Although there can be no certainties when eddy diffusivities are present, the use of representative parameters suggests that the eddy-induced circulation is indeed smaller than the wind-driven circulation in the Southern Ocean. That is, putting in numbers, we find $\alpha \approx 0.08$ with $\Psi^* \approx 1.6$ Sv and $\overline{\Psi} \approx 10$ Sv. This suggests that, for typical oceanic parameters, the strength of the eddy-induced circulation on isopycnals corresponding to the middepth overturning cell is only about 10%–20% of the wind-driven circulation. Thus, rather than the residual circulation vanishing as is sometimes assumed, the middepth residual circulation is comparable to the wind-driven circulation and acts to pull O(10) Sv of deep water formed at high northern latitudes in the North Atlantic back up to the surface. As a result, the depth scale of stratification h is not linearly proportional to the wind stress τ, as one would obtain from the vanishing residual circulation argument with a simple eddy parameterization (3.4), but rather it scales with τ as $\tau^{1/2}$ and is dependent on Δb, which is the buoyancy range for isopycnals that are shared between the circumpolar channel and the isopycnal outcrop region in the Northern Hemisphere. Note that, using a scaling for h in (4.9), one can easily obtain a scaling for the zonal baroclinic transport of the Antarctic Circumpolar Current (ACC).

In summary, in the limit of weak diapycnal mixing, relevant to the present middepth ocean, the strength of the middepth overturning circulation is primarily determined by the Ekman transport in the Southern Ocean. The rest of the ocean is essentially forced to adjust and produce the amount of deep water demanded by the Ekman transport and the associated wind-driven upwelling in the Southern Ocean. For instance, during the transient adjustment, the Ekman transport in the circumpolar channel, in conjunction with the surface buoyancy flux, pulls dense waters up from the deep ocean, converts them into light waters at the surface, and pumps these waters into or just below the main thermocline in the ocean basin. The rate at which these light waters are then imported into the deep-water formation region in the North Atlantic, converted back into dense waters, and exported to the ocean basin at middepth is controlled by the meridional pressure gradient set up by the outcropping isopycnals in the north. Hence, light waters pumped into the ocean basin by the Ekman transport in the south accumulate in or just below the main thermocline deepening therefore the middepth isopycnals and increasing the transport of light water into the deep-water formation region in the north until the transports in the north and south match. The established interhemispheric balance sets the depth of the isopycnals in the ocean basin and thus stratification throughout the entire ocean.

In the case when deep waters are not produced in the north, as observed in the Pacific Ocean, light waters pumped into the ocean basin by the Southern Ocean wind will deepen the middepth isopycnal in the ocean basin and thus steepen their slopes in the Southern Ocean, until the eddy-induced circulation in the Southern Ocean cancels the wind-driven circulation, resulting in a zero residual circulation and water mass transformation.

2) Strong diffusivity: Mixing-driven limit

Let us now consider a limit when the wind-driven upwelling in the Southern Hemisphere Ψ_1 is small compared to the mixing-driven upwelling in the ocean basin Ψ_2. This limit does not seem to be relevant for the present middepth ocean but could be relevant for a glacial climate, when the Southern Ocean winds might have been weaker (Toggweiler et al. 2006) and/or interior mixing might have been stronger (Egbert et al. 2004). In this limit, there is a balance between the deep-water formation at high latitudes in the Northern Hemisphere Ψ_3 and diapycnal upwelling in the ocean basin at low and middle latitudes Ψ_2,

$$\frac{\Delta b h^2}{f_0} = \frac{\kappa_v}{h} L_x L_y. \tag{4.11}$$

Solving this equation for the depth scale h, we obtain

$$h = \kappa_v^{1/3} \left(\frac{f_0}{\Delta b}\right)^{1/3} (L_x L_y)^{1/3} \approx 170\,\mathrm{m}, \tag{4.12}$$

which corresponds to a scaling for the rate of the overturning circulation,

$$\Psi_3 = \Psi_2 = \kappa_v^{2/3} \left(\frac{\Delta b}{f_0}\right)^{1/3} (L_x L_y)^{2/3} \approx 3\,\mathrm{Sv}. \tag{4.13}$$

These scalings are similar to the classical diffusive thermocline scalings (Welander 1971; Vallis 2006), and the mixing-driven overturning circulation estimated for the typical oceanic parameters is rather small. However, if an order of magnitude greater diapycnal mixing is used, then $h = 370$ m and $\Psi_3 = 13$ Sv. This limit is also relevant to early numerical studies of the MOC in a single Northern Hemisphere domain with a wall at the equator (e.g., Bryan 1991), where the cross-isopycnal mixing-driven upwelling is the only pathway for the deep waters to return back to the surface.

In this limit, both stratification and overturning circulation are maintained by the surface buoyancy distribution in the North Atlantic and the interior diapycnal mixing in the ocean basin. Physically, the maintenance of stratification is similar to that in the wind-driven limit, except that the role of the Ekman transport and surface buoyancy flux to convert deep dense waters into light waters in the circumpolar channel and pump them into the ocean basin is taken by diapycnal mixing. Diapycnal mixing fluxes heat from the warm upper ocean into the cold deep ocean, converts dense waters into light waters, and lets buoyancy forces lift them up. This process deepens isopycnals in the ocean basin until the rate of the dense-to-light water transformation in the basin interior matches to the rate of the light-to-dense transformation at high latitudes in the North Atlantic.

b. *Numerical solutions*

We now describe the numerical solver of the governing equations that we call the Semianalytical Model of the Bottom and Upper Cell of the Atlantic (SAMBUCA). Directly solving the equations of the theory numerically allows us to explore solutions in a broad range of parameters and enables a more direct comparison to be made with simulations using an ocean general circulation model, as described later.

The equations are solved by a forward integration of time-dependent form of the buoyancy advection–diffusion Eqs. (3.1) and (3.7) using the appropriate boundary conditions and the advective streamfunctions (3.2)–(3.4) and (3.11). The equations are discretized using finite differencing on a staggered Cartesian grid in the domain of the circumpolar channel, which extends from the southern boundary at $y = -2000$ km to the northern edge of the channel at $y = 0$ and from the bottom at $z = -4$ km to the surface at $z = 0$ with the meridional and vertical grid spacings of 100 km and 50 m, respectively. The region of the ocean basin where buoyancy varies only in the vertical is represented by a single grid cell in the meridional direction. The dimensions of the ocean basin are $L_x = 5000$ km and $L_y = 12\,000$ km. A uniform value of diapycnal diffusivity of $\kappa_v = 2 \times 10^{-5}$ m^2 s^{-1} is used in the control calculation. The eddy streamfunction in (3.4) is computed using a Gent–McWilliams type parameterization with the eddy diffusivity of $K_e = 1000$ m^2 s^{-1} in the form of the boundary-value problem (Ferrari et al. 2010), which allows for a smooth transition of the eddy streamfunction to the weakly stratified regions. In all the cases we report the solver converges to a steady state. Although numerical solutions can be found for almost arbitrary shapes of the surface boundary conditions and problem parameters, here we discuss solutions obtained for the surface buoyancy distributions and wind stress similar to the ones used in simulations with a GCM.

5. Comparison with general circulation model

In this section, we compare theoretical solutions and scalings with the results obtained from an idealized simulation with an ocean GCM.

a. *Zonally averaged sections*

Zonally averaged sections of the residual circulation and temperature from both GCM simulations and

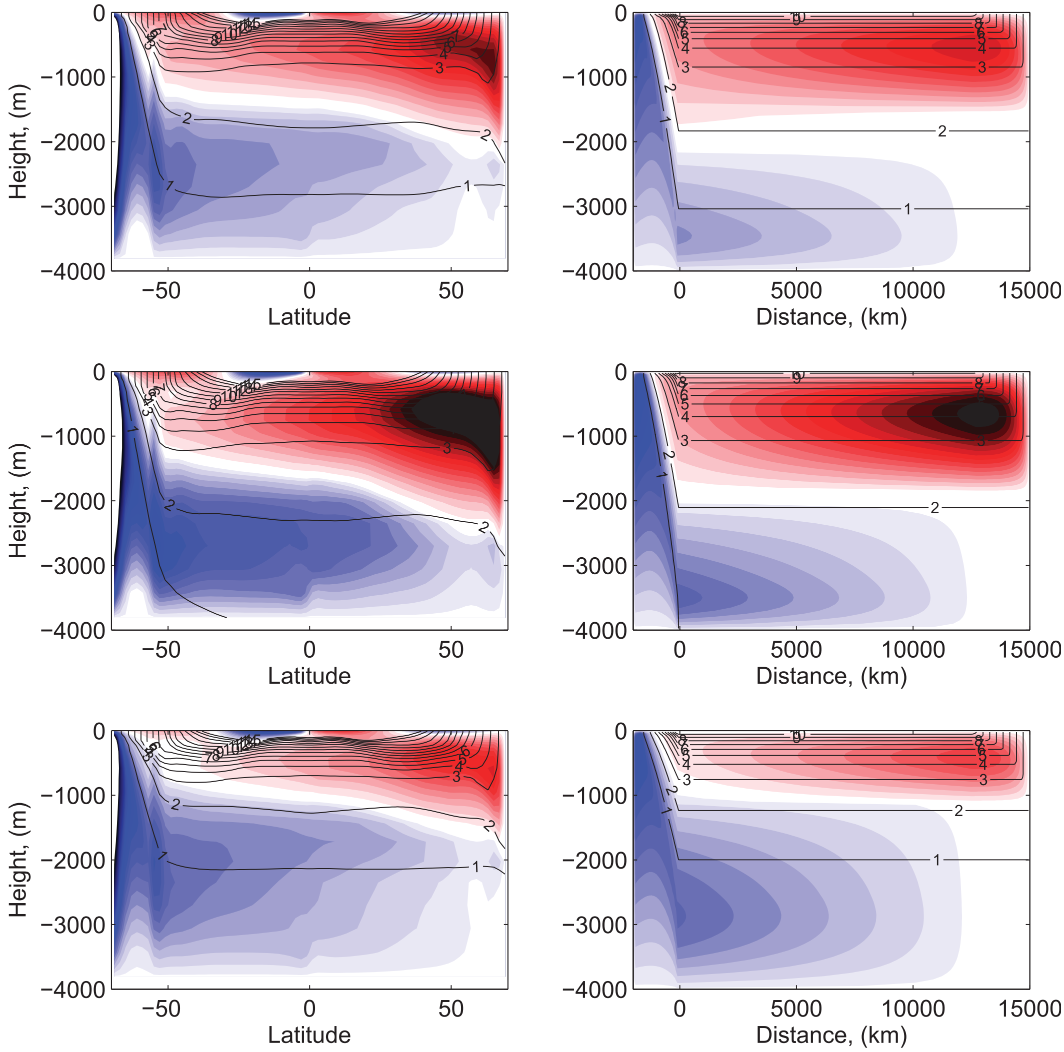

FIG. 9. Zonally averaged sections of residual circulation (Sv; blue/red) and temperature (°C, black lines) from (left) the GCM simulations and (right) theory for (top) the control experiment with $\kappa_v = 2 \times 10^{-5}$ m^2 s^{-1} and $\tau = 0.2$ N m^{-2}, (middle) an enhanced mixing experiment with $\kappa_v = 5 \times 10^{-5}$ m^2 s^{-1}, and (bottom) a reduced Southern Ocean wind experiment with $\tau = 0.1$ N m^{-2}.

theory are shown in Fig. 9. We show solutions from the control simulation, using $\kappa_v = 2 \times 10^{-5}$ m^2 s^{-1} and $\tau = 0.2$ N m^{-2}, as well as from the two perturbation experiments corresponding to enhanced diapycnal mixing, $\kappa_v = 5 \times 10^{-5}$ m^2 s^{-1}, and reduced Southern Ocean wind, $\tau = 0.1$ N m^{-2}. Although theoretical solutions are highly simplified, the theory seem to agree well both qualitatively and, as we show below, quantitatively with the GCM simulations.

In all experiments, there are two overturning cells in the deep ocean, away from the surface wind-driven circulation: the middepth and the abyssal cells. In the enhanced mixing experiment, isopycnals extend deeper into the ocean and the overturning rate of both cells is stronger than in the control experiment. We note that enhanced diapycnal mixing affects not only the density structure in the ocean basin away from the channel but also the density structure in the channel, with the slope of isopycnals in the circumpolar channel steeper in the enhanced mixing experiment. Hence, the enhanced mixing in the ocean basin not only increases the mixing-driven upwelling at low and middle latitudes but also decreases the wind-driven upwelling in the circumpolar channel by increasing isopycnal slope and therefore the rate of the eddy-induced circulation, which opposes the wind-driven upwelling.

In the reduced wind experiment, isopycnals are shallower than in the control experiment. The middepth

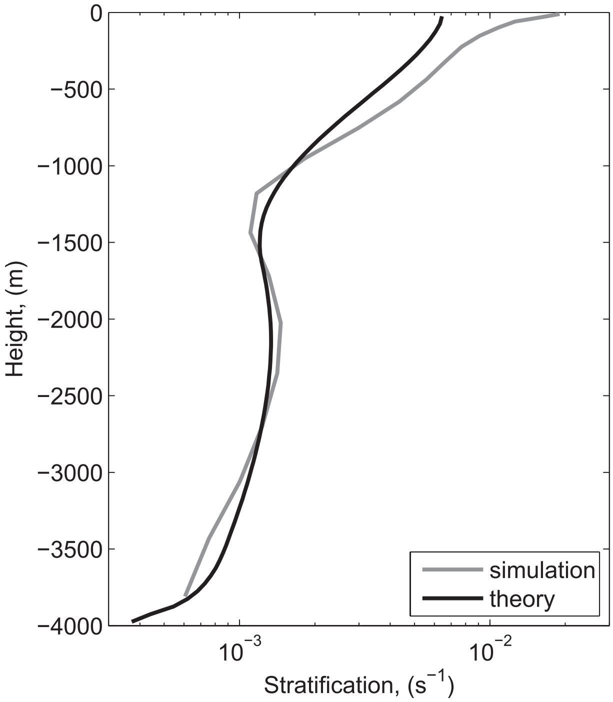

FIG. 10. Vertical profiles of stratification (s^{-1}) estimated from the idealized GCM simulation (gray) as an average over a low-latitude region from 30°S to 30°N and from the theory (black).

overturning cell is also significantly shallower and weaker with most of its water upwelling diffusively at low and middle latitudes. The middepth residual circulation in the circumpolar channel is nearly zero, corresponding to a cancellation between the wind-driven and eddy-induced circulations. The abyssal cell, on the other hand, becomes larger and somewhat stronger than in the control simulation. Isopycnals corresponding to the abyssal cell are, in general, steeper than isopycnals corresponding to the middepth cell resulting in the stronger eddy-induced circulation that fully cancels the wind-driven circulation and also drives the negative residual circulation, which matches to the mixing-driven upwelling in the ocean basin (Nikurashin and Vallis 2011).

b. Deep stratification

In Fig. 10, we compare the vertical profiles of stratification in the ocean basin estimated from the control simulation as an average over a low-latitude region from 30°S to 30°N, with theoretical prediction estimated from $b_2(z)$. Results show that both the idealized simulation and theory produce realistic stratification of $O(10^{-3})$ s^{-1} in the deep ocean. Stratification in the simulation increases by an order of magnitude to $O(10^{-2})$ s^{-1} in the upper kilometer or so. Theoretical prediction, however, increases only by about a factor of 5 and, in the upper ocean, is generally lower than stratification in the simulation. The reason for the lower stratification in the upper ocean in the theory is that the theory does not account for the high surface temperatures at low and middle latitudes that are present in the simulations. Also, stratification in the thermocline is maintained not only by the high-latitude Southern Ocean and North Atlantic processes, as assumed in the theory, but also by the local low-latitude and midlatitude winds (e.g., Luyten et al. 1983).

Both the idealized simulation and theory show the presence of a thermostad at middepth, at 1–2 km, where stratification has a local minimum resulting from an injection and southward propagation of the NADW. Stratification in the abyssal ocean, at 2.5–4-km depth, is well captured by the theory. Isopycnals at this depth outcrop only in the circumpolar channel and are too deep to be affected by the low-latitude and midlatitude winds. Thus, stratification at this depth is controlled primarily by the circumpolar channel and abyssal ocean processes (Nikurashin and Vallis 2011), which are similar between idealized simulations and theory.

c. Parameter sensitivity

We now quantitatively describe the dependence of the numerical solution on the parameters of the problem and in particular the transition of the solution from the adiabatic wind-driven to the diabatic mixing-driven circulation limits. We test the analytic scalings for the rate of the overturning circulation and stratification depth scale described in the previous section against numerical solutions computed for different values of diapycnal diffusivity κ_v and wind stress τ.

Figure 11 shows the rate of the overturning circulation and the depth scale of stratification as a function of the Southern Ocean wind stress and diapycnal diffusivity from both GCM simulations and theory. In the simulations, the rate of the overturning circulation is diagnosed as a Northern Hemisphere maximum of the overturning streamfunction averaged zonally along isopycnals. The depth scale of stratification is diagnosed as a depth of the 5°C zonally averaged isotherm at low latitudes between 30°S and 30°N. In the theory, the characteristic rate of the overturning circulation is calculated as a maximum of $\psi_3(z)$ and the stratification depth scale is estimated as a depth of 4°C isotherm from $b_2(z)$. The 4°C isotherm is used instead of the 5°C to eliminate the $O(100)$ m mismatch in the isopycnal depth due to the lower stratification in the upper ocean in the theory and make the curves collapse.

There is a good agreement between the theory and simulations. Both the theory and simulations are

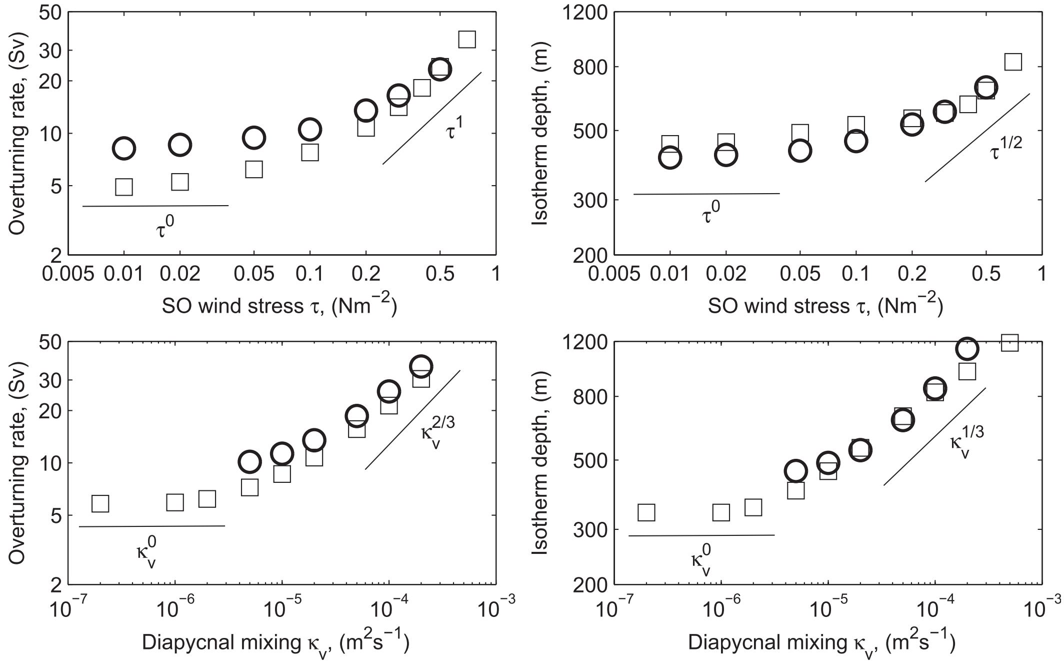

FIG. 11. Rate of (left) the overturning circulation (Sv) and (right) depth scale of stratification (meters) as a function of the Southern Ocean (top) wind stress and (bottom) diapycnal diffusivity estimated from the idealized GCM simulations (circles) and from the theory (squares).

characterized by two regimes corresponding to the wind-driven and mixing-driven circulations, respectively, with a rather smooth transition between the two. In the limit of weak diapycnal mixing or strong Southern Ocean wind, the rate of the overturning circulation becomes independent of diapycnal mixing and scales with the Southern Ocean wind stress as τ^1. The depth scale of stratification is also independent of mixing and scales with wind stress as $\tau^{1/2}$. This limit corresponds to the wind-driven circulation limit discussed above and scalings are consistent with (4.5) and (4.7). The nonlinear scaling of the stratification depth scale with the wind stress results from an interhemispheric equilibration between the Northern Hemisphere sinking, which scales quadratically with the stratification depth scale, and the Southern Ocean upwelling, which to the leading order is independent of the stratification depth scale.

On the other hand, in the limit of strong diapycnal mixing or weak Southern Ocean wind, the rate of the overturning circulation and the depth scale of stratification become independent of the wind stress and scale with diapycnal mixing as $\kappa_v^{2/3}$ and $\kappa_v^{1/3}$, respectively. This limit corresponds to the mixing-driven circulation limit and scalings are consistent with (4.13) and (4.12) and with previously obtained classical scalings described, for example, in Vallis (2006).

The control simulation parameters, typical for the present middepth ocean, are in the transition zone closer to the wind-driven circulation limit. The results suggest that a significant change in climate resulting in a reduction of the Southern Ocean wind and/or an increase in the ocean interior mixing can shift the MOC from the wind-driven to the mixing-driven regime characterized by a different sensitivity to external parameters.

6. Conclusions

We have presented a theory of the interhemispheric meridional overturning circulation and associated deep stratification in a single-basin ocean with a circumpolar channel. The theory relies on a matching of the dynamics among three regions of the ocean: the circumpolar channel in the Southern Hemisphere, a region of isopycnal outcrop in the Northern Hemisphere, and the enclosed ocean basin between them. In each region, the theory is based on the primitive equations of motion, but with different dominant balances, and includes the effects of wind, eddies, and diapycnal mixing. It explicitly predicts the deep stratification in terms of the surface forcing and other parameters of the problem and agrees well with a coarse-resolution ocean general circulation model configured in an idealized single-basin domain; that is, the parameter dependencies predicted by the theory are obeyed by the GCM.

The theory and idealized simulations show that, in the parameter range typical for the present climate ocean (i.e., a small diffusivity limit), the middepth interhemispheric overturning circulation is primarily driven by the Southern Ocean wind. The rate of the overturning circulation is essentially fixed by the Ekman transport in the Southern Ocean and therefore scales linearly with the wind stress τ. The circulation at high latitudes in the Northern Hemisphere is therefore forced to adjust to produce the amount of deep water demanded by the wind-driven upwelling in the Southern Ocean. The adjustment between the deep-water formation in the north and the wind-driven upwelling in the south sets the middepth stratification throughout the ocean. The rate of the deep-water formation in the north depends on the inflow of warm water from the ocean basin, which is in thermal balance with the buoyancy gradient created by the outcropping isopycnals. As a result of the interhemispheric adjustment the depth scale of stratification scales with the wind stress τ as $\tau^{1/2}$.

A reduction of the Southern Ocean winds and/or increase in the ocean interior mixing could shift the dynamics of the MOC from the wind-driven to mixing-driven limit. The dynamics then become more similar to a classical advective–diffusive balance and, consistent with classical diffusive thermocline scalings, the strength of the MOC and the depth scale of stratification scale with diapycnal diffusivity κ_v as $\kappa_v^{2/3}$ and $\kappa_v^{1/3}$, respectively. However, unless the diapycnal diffusivity is unrealistically large, the balance in the Southern Ocean reentrant channel is between mesoscale eddy effects and wind, with diffusive effects remaining small.

The depth scale of stratification, in both the high and low diffusivity limits, is inversely proportional to Δb, the buoyancy range of isopycnals shared between the circumpolar channel and the region of isopycnal outcrop in the North Atlantic. Thermal wind balance implies that, the greater the Δb, the shallower the isopycnals need be in the ocean basin h in order to match a given amount of wind-driven upwelling in the Southern Ocean.

Overall, the theory (and our simulations) appear to be consistent with the phenomenology described by Wolfe and Cessi (2011) using a conceptual model (in which eddies are parameterized) and numerical simulations (in which mesoscale eddies are resolved). They also find that a pole-to-pole circulation can be maintained if there is wind forcing over the southern circumpolar channel and if there is a set of isopycnals that outcrop in both the channel and the enclosed Northern Hemisphere basin. If the diapycnal diffusivity is small enough, the pole-to-pole circulation becomes essentially adiabatic, similar to the circulation described in section 4a.

There are a number of limitations to the study. One is the use of highly idealized geometry that includes neither a topographic ridge across the circumpolar channel to better represent the Drake Passage nor a midocean ridge to represent the Mid-Atlantic Ridge. In our simulations, we did find that ridges in the deep ocean have no major effect on the steady-state circulation (Fig. 5), but the impact of topography on the parameter sensitivity remains to be addressed. Another limitation is the single-basin configuration used in this study. Finally, eddies in our circumpolar channel are represented by a rather simple diffusive parameterization, albeit in a transformed Eulerian-mean framework to maintain adiabaticity. Almost certainly this is an imperfect parameterization of geostrophic turbulence. Our theory could in principle be modified if an improved parameterization were available, but we have not explored this. In spite of these limitations, the theory can be used to interpret basic changes in stratification and overturning circulation in comprehensive climate and ocean models and as a way to explore changes in circulation that may occur on very long time scales, such as the glacial/interglacial climate problem.

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