will restore symmetry, and the hysteresis characteristic will be the same as if it were treated by an ac field to 120°C. If the sample, after treatment, is heated to 400°C, without field applied, for one hour, it does not revert to its original state.

This modification is not permanent as can be seen in Fig. 2, where the decay is shown to occur in some hundreds of hours and

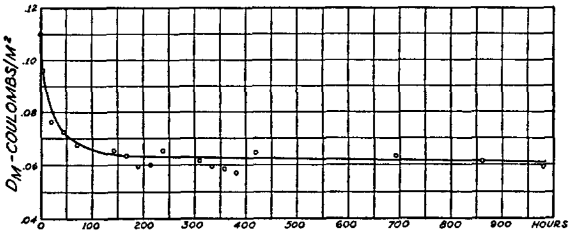

FIG. 2. Maximum D as a function of time. Field applied $=8.34\times10^5$ volts per meter.

varies from one sample to another. No dependence of decay rate on temperature has been found.

The author is indebted to Dr. Hans Jaffe of The Brush Development Company for calling his attention to the work of Rzhanov.

[1] A. V. Rzhanov, Zhur. Eksp. Teoret. Fiz. 19, 335–45 (1949).

Difference between Turbulence in a Two-Dimensional Fluid and in a Three-Dimensional Fluid

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THE difference between a two-dimensional and a three-dimensional fluid can easily be seen from the vorticity equation given as

$$\dot{\boldsymbol{\omega}}+(\mathbf{v},\nabla)\boldsymbol{\omega}=\nu\Delta\boldsymbol{\omega}+(\boldsymbol{\omega},\nabla)\mathbf{v}, \tag{1}$$

where $\boldsymbol{\omega}$, $\mathbf{v}$, and ν are the vorticity, velocity, and kinematic viscosity of the fluid. In the two-dimensional case, $(\boldsymbol{\omega},\nabla)\mathbf{v}$ is identically zero. Hence, if one neglects viscosity in a system moving with the fluid, the vorticity never changes, and the scattering of energy between eddies does not lead to any change in vorticity. This conservation law forbids the fulfillment of an ergodic hypothesis for a two-dimensional fluid. Indeed, if one raises the same arguments to a two-dimensional case, as in the current theory of statistical turbulence,[1] a contradiction can readily be found.

On multiplying Eq. (1) by $\boldsymbol{\omega}$ and transforming it to the wave number space, we can write

$$-\frac{\partial}{\partial t}\int_0^\infty F(k)k^2dk=2\nu\int_0^\infty F(k)k^4dk, \tag{2}$$

where $F(k)$ is the energy spectrum of a two-dimensional turbulence with k as the wave number. For definiteness, we shall use Heisenberg's[2] form of energy transfer between eddies which, for a steady state of turbulence, yields the form of $K(k)$ as

$$F(k)=\tfrac{1}{2}v_0^2k_0^{2/3}k^{-5/3}\quad\text{for}\quad k_0\leqslant k<k_s, \tag{3}$$

and

$$k_s/k_0=0.22(R_0\kappa)^{3/4},$$

where κ is the coefficient of eddy viscosity, and R_0 is the Reynolds' number given as $\pi v_0/\nu k_0$.

For a steady state of turbulence, we have

$$-\frac{\partial}{\partial t}\int_0^\infty F(k)k^2dk=-\frac{\partial}{\partial t}\int_0^{k_0} F(k)k^2dk<k_0^2\epsilon, \tag{4}$$

where ϵ equals the total energy supply per unit volume to the fluid and is given as[2]

$$\epsilon=\sqrt{3}k_0v_0^3\kappa/8. \tag{5}$$

On the other hand, the right-hand side of (2) is always greater than

$$2\nu\int_{k_0}^{k_s}F(k)k^4dk\cong\frac{\nu}{5}v_0^2k_0^{2/3}k_s^{10/3}. \tag{6}$$

Hence,

$$\frac{-\dfrac{\partial}{\partial t}\displaystyle\int_0^\infty F(k)k^2dk}{2\nu\displaystyle\int_0^\infty F(k)k^4dk}<56R_0^{-3/2}\kappa^{-3/2}. \tag{7}$$

We can easily see that (7) is in direct contradiction to (2). Since (7) is obtained by using only the behavior of $F(k)$ in the Kolmogoroff region ($F(k)\propto k^{-5/3}$), this contradiction will hold as long as one assumes the existence of a Kolmogoroff region for two-dimensional turbulence.

In the three-dimensional case, the vorticity may be changed by the extension of the vortex filament, and the application of the vorticity equation does not lead to any contradiction.[3]

The author wishes to thank Professor W. Heisenberg for an illuminating communication.

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[2] W. Heisenberg, Z. Physik 124, 628 (1948).
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The Generality of Mixed Gas Flows

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W. R. SEARS, in the August, 1950, issue of this Journal, speaks out in favor of neighbors, but still seems to reserve some doubts about the existence of neighbor solutions of mixed flows; that is, of solutions for neighbor contours. The following argument should convince him that neighbors are positively the rule, particularly with plane two-dimensional flows.

These flows are linear in the hodograph and, therefore, admit these superpositions. In the physical plane, the composite flow assigns, to each velocity vector, the center of gravity of the points occupied by the same vector in the constituent flows. Now superpose thus to the mixed flow a weak secondary constituent flow single valued through the pertinent portion of the hodograph. All velocities of the secondary flow taking part in the superposition will be contained within a small area in the physical plane. An immense variety of such secondary flows is available. Each gives rise to a different neighbor solution with the composite contour as close to the initial contour as desired.

The composite flow will be free of limiting lines in general. The discriminant, which becomes zero at the limiting line, is linear in the hodograph. (See reference 8 of Sears.) It is related to the angle between the streamline image and the characteristic line in the hodograph. This discriminant can be supposed to be larger than zero throughout the flow and also through a strip within a certain distance from the contour. The secondary constituent discriminant is finite and small, and by making the secondary constituent smaller and smaller the composite discriminant may be kept from becoming zero.

The composite flow will include an unbroken contour. For not only the legendre reciprocal potential but also the stream function is linear in the hodograph plane. The two impact points, having initially equal velocity and equal stream function, will, therefore, be transformed into two composite impact points having equal stream function.